Interbank Trade, Liquidity Reallocation and Welfare in a Monetary Economy*

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Abstract

We study positive and normative implications of interbank trade in a monetary economy in which banks require liquidity to finance stochastic investment opportunities and meet bilaterally to offset liquidity imbalances once these opportunities are realized. Depending on the nominal interest rate, the economy exhibits abundant- or scarce-liquidity equilibria, with distinct implications for aggregate investment and monetary transmission. In terms of welfare, we show analytically that when interest rates are low, either full participation or a complete market shutdown is optimal, depending on the bargaining power of banks with investment opportunities. However, when bargaining power is sufficiently high and interest rates rise above a threshold, it becomes optimal to restrict—though not fully shut down—participation in the interbank market.

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1 Introduction

The interbank market is an informal market where banks with different liquidity needs exchange reserves, mostly via uncollateralized, over-night debt. It provides a way to address cash shortages, without resorting to central bank lending facilities. The prevailing view in policy circles is that disruptions to interbank lending can be detrimental to credit provision by impairing banks' funding capacity. In this paper, we formally examine whether the ability to trade liquidity bilaterally indeed enhances credit provision, or whether it may have the opposite effect, within the framework of a monetary model that explicitly accounts for the over-the-counter (OTC) nature of the interbank market.

We start by developing a monetary economy in which banks bilaterally trade liquid assets in a decentralized market with search and bargaining.[1] Banks face random investment opportunities. Some become productive and gain access to a technology that requires capital goods, which must be purchased from suppliers in exchange for money, while other banks remain unproductive. To finance these opportunities, banks accumulate a buffer of liquid assets in anticipation of future investment opportunities. However, because inflation makes liquidity costly to hold, banks do not carry sufficient cash to invest efficiently. As a result, once uncertainty about investment opportunities is resolved, productive banks value additional liquidity, while unproductive banks hold idle cash. Banks can then rely on the interbank market to reallocate liquidity and offset these imbalances through unsecured monetary loans.

Upon a match, a productive bank borrows liquidity from its unproductive counterpart, and its investment decision depends on the joint cash holdings of the two parties. If a bank remains unmatched, it must rely solely on its own liquidity to purchase capital goods. Depending on the level of the nominal interest rate—which captures the opportunity cost of holding money—two types of stationary monetary equilibria can arise. When the interest rate is sufficiently low, the economy features an abundant-liquidity equilibrium, in which banks hold enough cash to invest efficiently, but only if they are able to find a counterpart from which to obtain the necessary

[1]Although our model is designed to capture the interbank market, our results extend to any uncollateralized over-the-counter (OTC) debt market.

liquidity. In contrast, when the interest rate is high, a scarce-liquidity equilibrium emerges, in which banks are liquidity constrained and unable to achieve their optimal investment levels, regardless of whether they are matched or not in the interbank market.

After characterizing the equilibrium of the model, we study how changes in the nominal interest rate and in market structure affect investment. As is standard in this class of models, an increase in the nominal interest rate induces banks to hold less liquidity, reducing capital purchases. The strength of this transmission depends on the efficiency of the interbank market and on the equilibrium regime. When the nominal interest rate is close to zero, higher trading probabilities weaken the transmission of interest rate changes; however, when the nominal interest rate is high and the economy operates in a scarce-liquidity equilibrium, greater trading efficiency instead amplifies the response.

Turning to market structure, we show that both individual and aggregate investment are lower when banks with investment opportunities can retain a larger share of the surplus in bilateral negotiations. The reason is that greater bargaining power by productive participants weakens incentives to hold liquidity. It makes banks more inclined to rely on liquidity obtained through the interbank market when an investment opportunity arises and, in addition, makes holding liquidity less attractive because unproductive counterparties can appropriate only a smaller share of the surplus. Finally, the effect of higher matching efficiency varies across equilibrium regimes and is generally ambiguous. Analytically, we identify two cases in which an increase in matching efficiency raises aggregate investment unambiguously: first, when the nominal interest rate is close to zero; and second, when productive banks' bargaining power is low and the nominal interest rate high enough for the economy to operate in a scarce-liquidity equilibrium.

Having characterized the relevant comparative statics, we next turn to the normative implications of the model and ask whether full participation, limited participation, or a complete shutdown of the interbank market is optimal in this economy. Changes in trading probabilities generate opposing welfare effects. On the one hand, a higher matching rate improves the reallocation of liquidity from unproductive to productive banks, which tends to raise aggregate investment and output. On the other hand, it also affects banks' incentives to accumulate liquidity ex ante, which may increase or reduce investment.

In an abundant-liquidity equilibrium, higher trading probabilities unambiguously decrease the value of holding money. The reason is twofold. First, banks can more easily rely on interbank liquidity to finance investment, reducing the value of carrying their own liquidity. Second, in an abundant-liquidity equilibrium only a fraction of an unproductive bank's money balances needs to be reallocated for a productive bank to be able to invest efficiently. As a result, the marginal liquidity held by an unproductive bank cannot be productively employed, and its value simply reflects the price of money.

In a scarce-liquidity equilibrium, the effect on the marginal value of money is ambiguous. Because aggregate liquidity is insufficient, this marginal value—conditional on being unproductive—reflects both the price of money and the additional surplus generated in a match. As in the abundant-liquidity case, higher matching efficiency weakens banks' incentives to hold costly liquidity, since productive banks can more easily rely on interbank credit. However, an opposite force arises: matched unproductive banks can fully lend out their idle liquidity and put it to productive use. Therefore, by increasing the probability of deploying idle cash, higher matching efficiency can strengthen incentives to accumulate liquidity.

In both equilibrium regimes, if the effect of higher matching efficiency on the value of holding money is sufficiently negative, the opportunity to trade can dry up aggregate liquidity. This in turn may lower investment and also increase the dispersion of marginal returns, potentially offsetting the benefits of liquidity reallocation. For this reason, it may be optimal to restrict participation or even fully shut down the interbank market. Analytically, we establish two main results. First, when the interest rate is sufficiently low so that the economy operates in an abundant-liquidity equilibrium, the optimal participation regime depends on the bargaining power of productive banks. If these banks capture a sufficiently large share of the surplus, a full market shutdown is optimal; otherwise, full participation is optimal. Second, when the bargaining power of productive banks exceeds a threshold and the interest rate is sufficiently high, it is optimal to restrict participation in the interbank market rather than to shut it down completely.

In the final part of the paper, we extend our model to allow for an exogenous limit on the amount of interbank debt that can be issued. We show that additional equilibrium types emerge in which the economy is debt constrained and banks borrow up to this limit. If unproductive banks retain a share of the surplus, there exists a non-empty range of interest rates for which an increase in the nominal interest rate has no effect on aggregate investment, while higher matching efficiency unambiguously raises aggregate investment. We also show that our welfare results continue to hold in this environment. In particular, there exists a range of low interest rates for which full market participation is optimal when the bargaining power of productive banks is sufficiently low, whereas a full market shutdown is optimal otherwise. Finally, as in the baseline model, we prove that when the interest rate becomes sufficiently high and liquidity sufficiently scarce, optimal participation in the interbank market is interior, and neither full participation nor a market shutdown is optimal.

2 Literature Review

This paper builds on an extensive literature studying search-theoretic models of money and bilateral financial trade, following the seminal work of Lagos and Wright (2005), as surveyed by Williamson and Wright (2010), Nosal and Rocheteau (2011), and Lagos et al. (2017). Within this line of research, our paper is closest to Rocheteau et al. (2018) and Bethune et al. (2022), who introduce search and matching frictions in settings where entrepreneurs engage in bilateral credit relationships with banks to finance investment opportunities.[2] As in these contributions, we develop a model in which money serves as a medium of exchange to finance capital purchases. However, while Rocheteau et al. (2018) and Bethune et al. (2022) focus on the role of real credit between banks and entrepreneurs, we study the implications of monetary interbank lending and liquidity reallocation for investment and welfare.

By modeling an interbank market in which banks meet bilaterally to trade liquidity, we connect our analysis to the broader literature on OTC financial markets, beginning with Duffie et al. (2005) and Ashcraft and Duffie (2007). Afonso and La-

[2]Our paper is also related to Wright et al. (2020), who study bilateral trade in capital goods among firms facing stochastic investment opportunities.

gos (2015) model the intraday federal funds market, emphasizing its OTC structure. While their analysis focuses on heterogeneity in banks' initial reserve positions, we study interbank trade in an environment where banks hold identical reserve balances that must be reallocated following the realization of stochastic investment opportunities.[3]

In addition, our analysis of optimal participation in the interbank market is related to several recent contributions, including Berentsen et al. (2014), Geromichalos and Herrenbrueck (2016), and Huber and Kim (2019). These papers study environments in which agents face idiosyncratic risk and invest in money and illiquid bonds that can be traded to obtain liquidity after the realization of shocks. By contrast, in our model banks directly reallocate liquidity through monetary credit arrangements in the interbank market. Our welfare analysis is also related to Berentsen et al. (2007), who show that welfare can be improved through monetary lending from liquidity-rich to liquidity-poor agents.[4] Relative to this paper, we show that monetary lending may reduce welfare when liquidity reallocation is bilateral and subject to frictions, as is the case in interbank markets. Our extension with debt limits is related to Gu et al. (2016), who study the real effects of credit conditions in an economy where money and credit coexist.

Our paper also relates to a large literature on monetary policy and the interbank market, including Poole (1968), Bech and Keister (2017), and Vari (2020). These models typically abstract from the OTC nature of interbank trading and emphasize the benefits of a well-functioning interbank market. Our approach is closer to Bech and Monnet (2016) and Bianchi and Bigio (2022), who explicitly model search frictions, though they do not discuss welfare implications. Finally, we contribute to an extensive literature that emphasizes transaction costs and asymmetric information as additional frictions in the interbank market (e.g. Clouse and Dow Jr (1999), Bartolini et al. (2001), Freixas and Jorge (2008), Heider et al. (2015)). We instead focus on fric-

[3] Within a Lagos and Wright (2005) framework, the effects of search frictions in models of bilateral financial trade have also been examined by Lagos (2011), Geromichalos and Herrenbrueck (2016), Mattesini and Nosal (2016) and Lagos and Zhang (2019).

[4] Ferraris and Mattesini (2020) show that money and debt collateralized by real assets can complement each other and improve allocations in an environment with informational and commitment frictions. Similarly, Araujo and Ferraris (2022) show that monetary loans can implement Pareto-superior allocations relative to real credit.

tions inherent to the OTC nature of interbank trading and abstract from asymmetric information and transaction costs.

3 Model

3.1 Environment

Time is discrete and is indexed by $t = 1, 2, 3, \ldots$. The economy features two goods: a perishable consumption good and a capital good, which is assumed to depreciate fully at the end of each period. In addition, there is an intrinsically useless asset, money, which is storable and whose supply evolves according to $M_{t+1} = \mu M_t$, with $\mu > 0$.[5] The economy is populated by two types of infinitely lived agents, each with unit mass: capital suppliers, who produce capital goods at cost $c(k) = k$, and banks, who may have the opportunity to put capital to productive use.[6]

As in Lagos and Wright (2005), a period consists of two successive sub-periods. In the second sub-period, all agent participate in a frictionless, centralized market where they can trade consumption goods and money. In the first subperiod, two markets are active concurrently: a competitive capital market and a decentralized interbank market. At the beginning of each period, banks face an idiosyncratic shock: with probability σ they have an investment opportunity. We refer to banks with an investment project as productive banks and banks without such opportunity as unproductive banks. The investment opportunity requires capital good k as input and yields $f(k)$ units of the consumption good at the beginning of the second sub-period. We assume $f(k)$ is homogeneous of degree $\alpha < 1$ and satisfies the Inada conditions. Agents are anonymous in the capital goods market, so suppliers require immediate compensation from banks—in the form of money—to provide capital goods.

Concurrently, there is an interbank market where banks with investment opportunities meet bilaterally with banks without. Following Berentsen et al. (2007), there

[5] Monetary expansions (contractions) are implemented via lump sum transfers to (taxes on) banks.

[6] Focusing on a linear cost of production allows us to isolate the effects of trading frictions in the interbank market. Similarly to Chiu et al. (2018), with a nonlinear cost of producing capital there is a pecuniary externality at play through the price of capital that introduces an additional source of inefficiency.

exists a technology that records banks' financial histories but does not track transactions in the capital goods market. This assumption rules out the use of real credit between productive banks and capital suppliers, but it allows banks to engage in monetary lending: when a productive and an unproductive bank meet, they may exchange money for claims on consumption goods to be settled between themselves in the second sub-period.[7] In our baseline model, we assume that banks can commit to repay interbank loans, or, equivalently, that repayment can be perfectly enforced. In Section 4 we relax this by imposing a limit on the amount of debt banks can credibly commit to pay back.

In this environment, gains from trade arise because, once idiosyncratic uncertainty is realized, banks wish to reallocate liquidity. Banks without investment opportunities no longer have a use for money, while banks with opportunities may value additional liquidity. Letting $\mathcal{M} \in [0, \min\{\sigma, 1-\sigma\}]$ denote the total number of matches in the interbank market, the matching rate for productive banks is $\eta = \frac{\mathcal{M}}{\sigma}$, and for unproductive banks is $\eta \frac{\sigma}{1-\sigma}$. Upon meeting, the counterparts negotiate the terms of the loan. We assume that terms of trade are determined by Kalai bargaining, with the productive bank holding bargaining power $\theta \in [0, 1]$.[8]

The preferences of banks are given by

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t c_t^b$$

where $c_t^b \in \mathcal{R}$ denotes net consumption in the second subperiod and $\beta \in (0, 1)$ the bank's subjective discount factor.[9] The preferences of capital suppliers are given by

$$\sum_{t=0}^{\infty} \beta^t c_t^s$$

[7]It is easy to show that capital suppliers would not participate in the interbank market even if they were given the opportunity. More importantly, we assume that the interbank market matches only banks of different types: those with investment opportunities act as borrowers, whereas those without act as lenders.

[8]In Appendix B, we show that our results continue to hold under Nash bargaining.

[9]The expectation operator, E, is with respect to the probability measure induced by the random arrival of investment opportunities and by the matching process in the interbank market.

where $c_t^s \in \mathcal{R}_+$ denotes consumption in the second subperiod.[10]

3.2 Capital Suppliers

A capital supplier enters the second sub-period with money holdings m_t^s, uses them entirely for consumption, and carries no money into the next period.[11] Letting ϕ_t denote the price of money in terms of consumption goods, the value function of a supplier in the second sub-period is

$$W_t^s(m_t^s) = \phi_t m_t^s + \beta V_{t+1}^s$$

where V_{t+1}^s denotes the value function at the beginning of the next period.

In the first sub-period, a supplier participates only in the capital market and chooses how much capital to produce. The beginning-of-period value function is therefore

$$V_t^s = \max_{k_t^s} \left\{ -c(k_t^s) + W_t^s\left(\frac{p_t k_t^s}{\phi_t}\right) \right\}$$

where p_t is the price of the capital good in terms of consumption goods. Since $W^s(m_t^s)$ is linear in m_t^s, the supplier's problem reduces to

$$\max_{k_t^s} \{-c(k_t^s) + p_t k_t^s\}$$

As $c(k_t^s) = k_t^s$, if the market is active, in equilibrium, it must be that:

$$p_t = 1 \tag{1}$$

3.3 Banks

Next, we turn to the problem of banks. In the second sub-period, banks choose how much to consume and how much money to carry into the next period. Let $W^P(m_t, d_t, k_t)$ denote the value function of a productive bank, where m_t is the money

[10] We assume that suppliers and banks share the same discount factor, β. In addition, we assume that capital suppliers cannot produce consumption goods.

[11] Since $\mu > \beta$, holding money is always costly. Because capital suppliers do not value money in the first sub-period, they optimally carry no money into the next period.

carried over from the first sub-period, k_t is the amount of capital, and d_t is the interbank debt position. A productive bank solves

$$W_t^P(m_t, d_t, k_t) = \max_{m_{t+1} \geq 0,\, c_t} \{c_t + \beta\, \mathbb{E}[V_{t+1}(m_{t+1})]\}$$

subject to

$$c_t + \phi_t m_{t+1} + d_t = f(k_t) + \phi_t(m_t + T_t),$$

where $\mathbb{E}[V_{t+1}(m_{t+1})]$ is the expected beginning-of-period value function, and T_t is the lump-sum monetary transfer from the central bank.

An unproductive bank enters the second sub-period with no capital, and its value function, $W^u(m_t, d_t)$, similarly solves

$$W_t^u(m_t, d_t) = \max_{m_{t+1} \geq 0,\, c_t} \{c_t + \beta\, \mathbb{E}[V_{t+1}(m_{t+1})]\}$$

subject to

$$c_t + \phi_t m_{t+1} + d_t = \phi_t(m_t + T_t),$$

Using the budget constraint and defining

$$W_t \equiv \phi_t T_t + \max_{m_{t+1} \geq 0} \left(-\phi_t m_{t+1} + \beta\, \mathbb{E}[V_{t+1}(m_{t+1})]\right),$$

we can express the value functions compactly as

$$W_t^P(m_t, d_t, k_t) = f(k_t) + \phi_t m_t - d_t + W_t,$$
$$W_t^u(m_t, d_t) = \phi_t m_t + d_t + W_t.$$

As in Lagos and Wright (2005), these expressions imply that choices made in the centralized market are independent of a bank's prior history; hence all banks enter the next period with the same money holdings m_t. Assuming an interior solution, the optimality condition for money holdings is

$$\phi_t = \beta\, \frac{\partial \mathbb{E}[V_{t+1}(m_{t+1})]}{\partial m_{t+1}}. \tag{2}$$

At the beginning of the period, the preference shock is realized and each bank has an investment opportunity with probability σ. In addition, a productive bank has the opportunity to trade in the interbank market with probability η, while an unproductive bank has the opportunity to trade with probability $\eta\frac{\sigma}{(1-\sigma)}$. Letting a hat indicate that the bank is unmatched, the expected beginning-of-period value function is given by

$$\mathbb{E}[V_t(m_t)] = \sigma\left(\eta V_t^P(m_t) + (1-\eta)\hat{V}^P(m_t)\right) + (1-\sigma)\left(\eta\frac{\sigma}{(1-\sigma)}V_t^U(m_t) + \left(1-\eta\frac{\sigma}{(1-\sigma)}\right)\hat{V}_t^U(m_t)\right) \quad (3)$$

Here, $V^P(m_t)$, $V^U(m_t)$ $\hat{V}^P(m_t)$ and $\hat{V}^U(m_t)$ denote the value functions for a matched productive bank, a matched unproductive bank, an unmatched productive bank, and an unmatched unproductive bank, respectively.

An unmatched unproductive bank is inactive in the first sub-period, and its value function is simply.

$$\hat{V}_t^U(m_t) = W_t^u(m_t, 0) = \phi_t m_t + W_t \quad (4)$$

For an unmatched productive bank, the value function is the solution to the following problem

$$\hat{V}_t^P(m_t) = \max_{\hat{k}_t} W_t^P\left(m_t - \frac{\hat{k}_t}{\phi_t},\, 0,\, \hat{k}_t\right) = \max_{\hat{k}_t}\left\{f(\hat{k}_t) - \hat{k}_t + \phi_t m_t + W_t\right\} \quad (5)$$

subject to

$$m_t \geq \frac{\hat{k}_t}{\phi_t}.$$

This constraint states that an unmatched productive bank cannot spend more money than it brings from the previous period's centralized market. The bank's optimal choice of capital is given by

$$\hat{k}_t(m_t) = \begin{cases} k^*, & \text{if } \phi_t m_t \geq k^*, \\ \phi_t m_t, & \text{otherwise,} \end{cases} \tag{6}$$

where k^* solves $f'(k^*) = 1$ and thus corresponds to the efficient level of capital. This solution implies that an unmatched productive bank purchases the efficient amount of capital whenever its money holdings are sufficient; otherwise, it is liquidity constrained and spends all its money on capital.

Consider next a bilateral meeting in the interbank market between a productive bank with money holdings m_t^P and an unproductive bank with money holdings m_t^U. The terms of trade can be summarized by the triple (ψ_t, d_t, k_t). The unproductive bank transfers ψ_t units of real money balances in exchange for a promise of repayment d_t in consumption goods to be delivered in the second sub-period. The productive bank then uses her post-trade money holdings, $m_t^P + \psi_t$, to purchase k_t units of capital. If the parties fail to reach an agreement, they retain their pre-trade money balances, and the productive bank can still purchase $\hat{k}_t$ units of capital from suppliers. This implies that the surplus from the match for a productive bank is

$$\begin{aligned} \mathcal{S}^P &= W_t^P\left(m_t + \psi_t - \frac{k_t}{\phi_t}, d_t, k_t\right) - W_t^P\left(m_t - \frac{\hat{k}_t}{\phi_t}, 0, \hat{k}_t\right) \\ &= f(k_t) - k_t - d_t + \phi_t\psi_t - f(\hat{k}_t) + \hat{k}_t \end{aligned}$$

Similarly, the match surplus for an unproductive bank is:

$$\begin{aligned} \mathcal{S}^U =& W_t^U(m_t - \psi_t, d_t) - W_t^U(m_t, 0) \\ =& d_t - \phi_t\psi_t \end{aligned}$$

Assuming Kalai bargaining and letting $\theta \in [0, 1]$ be the share of the productive bank, leads to the following optimum problem.

$$(\psi_t, d_t, k_t) \in \arg\max_{\psi_t, d_t, k_t} \left\{ f(k_t) - d_t + \phi_t\psi_t - k_t - f(\hat{k}_t) + \hat{k}_t \right\} \tag{7}$$

subject to

$$f(k_t) - k_t - d_t + \phi_t\psi_t - f(\hat{k}_t) + \hat{k}_t = \frac{\theta}{1-\theta}\Big[d_t - \phi_t\psi_t\Big] \quad (8)$$

$$\frac{k_t}{\phi_t} \leq m_t^P + \psi_t \quad (9)$$

$$-m_t^P \leq \psi_t \leq m_t^U \quad (10)$$

The first constraint requires that a productive bank retains a fraction θ of the total surplus generated in the match, the second constraint is the liquidity constraint of a productive bank stating that it cannot spend more money than it holds, and the third constraint states that banks in a bilateral meeting can only reallocate money among themselves. The following lemma characterizes the solution to the bargaining problem

Lemma 1. *Let $(\psi_t(m_t^P, m_t^U), d_t(m_t^P, m_t^U), k_t(m_t^P, m_t^U))$ be the solution to the bargaining problem between a productive bank, with money holding m_t^P, and an unproductive bank, with money holding m_t^U.*

1. *If $m_t^P + m_t^U \geq \frac{k^*}{\phi_t}$, then*

$$k_t(m_t^P, m_t^U) = k^* \quad (11)$$

$$\psi_t(m_t^P, m_t^U) \in \left(\frac{k^*}{\phi_t} - m_t^P, m_t^U\right) \quad (12)$$

2. *If $m_t^P + m_t^U < \frac{k^*}{\phi_t}$, then*

$$k_t(m_t^P, m_t^U) = \phi_t(m_t^P + m_t^U) \quad (13)$$

$$\psi_t(m_t^P, m_t^U) = m_t^U \quad (14)$$

The interbank loan is

$$d_t(m_t^P, m_t^U) = \phi_t\psi_t(m_t^P, m_t^U) + (1-\theta)\big[f(k_t(m_t^P, m_t^U)) - k_t(m_t^P, m_t^U) - f(\hat{k}_t(m_t^P)) + \hat{k}_t(m_t^P)\big] \quad (15)$$

where $\hat{k}_t(m_t^P)$ is given by Equation 6.

The result is intuitive: the bargaining outcome in a bilateral meeting depends on the combined liquidity of the two parties. If the productive bank can obtain enough real balances from its counterpart, it escapes the liquidity constraint and attains the efficient level of investment. Conversely, if the constraint remains binding even when $\psi_t = m_t^U$, investment falls short of the first best. In both cases, for any given monetary transfer ψ_t, the interbank loan d_t is determined so that the surplus is shared between the parties according to their respective bargaining powers.

Equipped with the terms of trade, we can now write the first sub-period value functions for a bank that enters the period with money holdings m_t:

$$\begin{aligned} V_t^P(m_t) &= W_t^P\left(m_t + \psi_t(m_t, m_t^U) - \frac{k_t(m_t, m_t^U)}{\phi_t},\, d_t(m_t, m_t^U),\, k_t(m_t, m_t^U)\right) \\ &= \theta\big[f\big(k_t(m_t, m_t^U)\big) - k_t(m_t, m_t^U)\big] + (1-\theta)\Big[f\Big(\hat{k}_t(m_t)\Big) - \hat{k}_t(m_t)\Big] \\ &\quad + \phi_t m_t + W_t, \end{aligned} \tag{16}$$

$$\begin{aligned} V_t^U(m_t) &= W_t^U\big(m_t - \psi_t(m_t^P, m_t),\, d_t(m_t^P, m_t)\big) \\ &= (1-\theta)\Big[f\big(k_t(m_t^P, m_t)\big) - k_t(m_t^P, m_t) - f\Big(\hat{k}_t(m_t^P)\Big) + \hat{k}_t(m_t^P)\Big] \\ &\quad + \phi_t m_t + W_t. \end{aligned} \tag{17}$$

Here, m_t^U denotes the money holdings of an unproductive counterpart, while m_t^P denotes the money holdings of a productive counterpart.[12]

3.4 Equilibrium

Having characterized the decisions of banks and suppliers, we can now define the competitive equilibrium in this economy.

Definition 1. *Given a money growth rule μ and an initial money supply M_0, an equilibrium consists of i) a set of prices $\{\phi_t, p_t\}_{t=0}^{\infty}$, ii) allocations in the interbank*

[12] Here we have used the fact that all banks of a given type—productive or unproductive—enter the period with the same amount of cash. In fact, in equilibrium, money holdings are also equalized across types, so that productive and unproductive banks carry the same amount of cash.

market $\{\psi_t, d_t\}_{t=0}^{\infty}$*, iii) allocations in the capital market* $\{k_t, \hat{k}_t\}_{t=0}^{\infty}$*, iv) consumption paths* $\{c_t^b, c_t^s\}_{t=0}^{\infty}$*, and v) end-of-period money holdings* $\{m_t\}_{t=0}^{\infty}$ *such that for all* t *the optimality conditions of banks and capital suppliers hold and all markets clear.*

The next proposition provides a complete characterization of the equilibrium.

Proposition 1. *A path for prices* $\{\phi_t\}_{t=0}^{\infty}$*, allocations in the competitive capital market* $\{\hat{k}_t, k_t\}_{t=0}^{\infty}$ *and end-of-period money holdings* $\{m_t\}_{t=0}^{\infty}$ *constitute an equilibrium if and only if they satisfy the following conditions:*

1. *Optimal allocations in the competitive capital market*

$$\hat{k}_t = \hat{k}_t(m_t) \equiv \begin{cases} k^*, & \text{if } k^* \le \phi_t m_t \\ \phi_t m_t, & \text{otherwise} \end{cases}$$

$$k_t = k_t(m_t) \equiv \begin{cases} k^*, & \text{if } k^* \le 2\phi_t m_t \\ 2\phi_t m_t, & \text{otherwise} \end{cases}$$

2. *Optimal end-of-period money holding*

$$\frac{\phi_t}{\beta} = \phi_{t+1} + \sigma\eta\frac{1}{2}\frac{\partial k_t(m_{t+1})}{\partial m_{t+1}}[f'(k_{t+1}(m_{t+1})) - 1] + \sigma(1-\theta\eta)\frac{\partial \hat{k}_{t+1}(m_{t+1})}{\partial m_{t+1}}[f'(\hat{k}_{t+1}(m_{t+1})) - 1]$$

3. *Money market clearing*

$$m_t = M_t$$

We now focus on symmetric stationary equilibria in which the end-of-period real money balances are time-invariant $\phi_t M_t = \phi_{t+1} M_{t+1}$. Given the growth rate of money, this implies that $\frac{\phi_t}{\phi_{t+1}} = \mu$. Following the literature, we define i as the nominal interest rate on an illiquid, one-period bond. Using the Fisher equation, we can write $\frac{\mu-\beta}{\beta} = i$. In Appendix A we prove the following result

Proposition 2. *For any* $i > 0$*, there exists a unique stationary monetary equilibrium. There exists a threshold,* $\bar{i}$*, such that:*

1. If $i \leq \bar{i}$, there is an abundant-liquidity equilibrium satisfying:

$$k = k^* \tag{18}$$

$$i = \sigma(1-\theta\eta)\left(f'(\hat{k}) - 1\right) \tag{19}$$

2. If $i > \bar{i}$, there is a scarce-liquidity equilibrium satisfying:

$$k = 2\hat{k} \tag{20}$$

$$i = \sigma(1-\theta\eta)\left(f'(\hat{k}) - 1\right) + \sigma\eta\left(f'(2\hat{k}) - 1\right) \tag{21}$$

The threshold is given by:

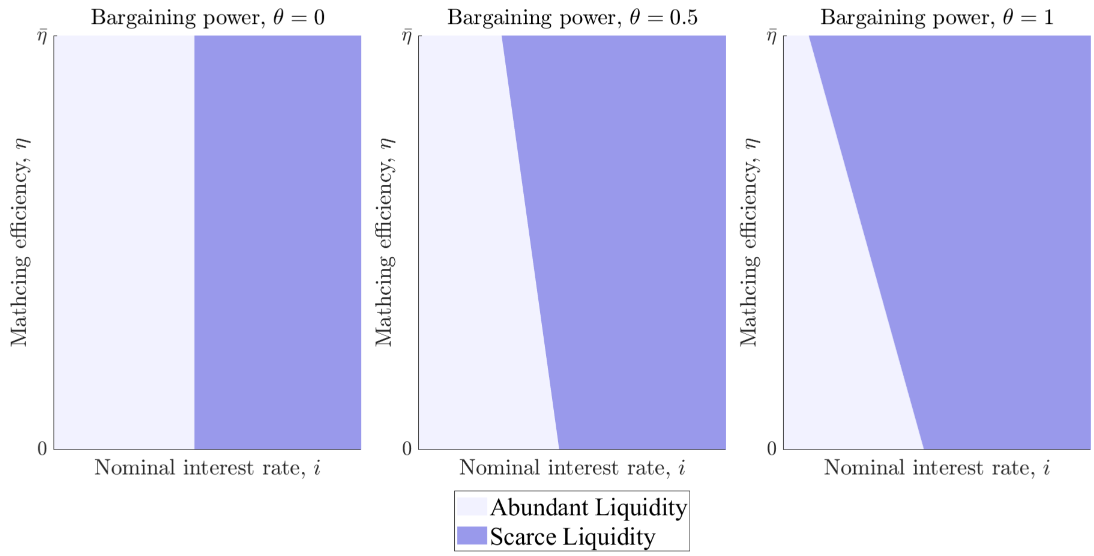
$$\bar{i} = \sigma(1-\theta\eta)\left(f'\left(\frac{k^*}{2}\right) - 1\right) \tag{22}$$

Proposition 2 shows that the economy features two types of equilibria, depending on the level of the nominal interest rate. In the *abundant-liquidity equilibrium*, the joint liquidity of productive and unproductive banks is sufficient to finance the efficient level of capital, k^*. In this case, by pooling real balances with their counterpart, productive banks are able to overcome the liquidity constraint. In the *scarce-liquidity equilibrium*, joint cash holdings are insufficient to purchase k^*, and even after pooling resources, capital falls short of the first-best level.

Proposition 2 further shows that the abundant-liquidity equilibrium arises when the interest rate is sufficiently low. Intuitively, when the cost of carrying money—captured by the nominal interest rate—is low, banks find it worthwhile to accumulate a large precautionary buffer of liquid assets. This improves the liquidity position of productive banks and also leads unproductive banks to hold substantial excess liquidity, which can be reallocated once idiosyncratic uncertainty is realized.

Figure 1 provides a graphical representation of the cutoff point for different values of the bargaining power parameter, θ. A lower bargaining power of productive banks, as well as a lower matching efficiency, both enlarge the range of interest rates for which the economy falls into the abundant-liquidity equilibrium.

Figure 1: Equilibria Cases



These figures show the parameter space for which an abundant liquidity equilibrium (light blue) and scarce liquidity equilibrium (dark blue) exists for different bargaining powers. This figure is created with $f(k) = k^{0.3}$ and $\sigma = 0.55$.

3.5 Comparative Statics

In this section, we conduct a series of comparative statics exercises to study the equilibrium effects of changes in key model parameters. We focus in particular on the effects of: 1) the nominal interest rate, i, which determines the cost of carrying money across periods; 2) the bargaining power, θ, which pins down the division of surplus between productive and unproductive agents; and 3) the matching efficiency, η, which affects the ability of banks to reallocate idle money and shapes their incentives to accumulate liquidity.[13]

The following proposition provides a characterization of the relevant comparative

[13]We consider the total number of bilateral meetings, $\mathcal{M}$, as a measure of the efficiency of the interbank market. If $\mathcal{M} = 0$, banks never match as if there was no interbank market. As the total number of bilateral meetings directly determines the probability of matching, we study comparative statics with respect to η in our analysis.

statics, if the economy is in an abundant-liquidity equilibrium:

Proposition 3. *For $i < \bar{i}$, we have*

1. $\frac{\partial k}{\partial i} = 0$, $\frac{\partial \hat{k}}{\partial i} < 0$, $\frac{\partial K}{\partial i} < 0$,
2. $\frac{\partial k}{\partial \theta} = 0$, $\frac{\partial \hat{k}}{\partial \theta} < 0$, $\frac{\partial K}{\partial \theta} < 0$,
3. $\frac{\partial k}{\partial \eta} = 0$, $\frac{\partial \hat{k}}{\partial \eta} < 0$, $\frac{\partial K}{\partial \eta} \gtrless 0$.

In addition, if $i \approx 0$, then $\frac{\partial K}{\partial \eta} > 0$

In an abundant-liquidity equilibrium, matched banks attain the unconstrained capital level k^*. This implies that, in this type of equilibrium, the capital k purchased by matched banks is unaffected by changes in the nominal interest rate, bargaining power, or matching efficiency. By contrast, the capital $\hat{k}$ of unmatched banks does respond to these parameters. Specifically, we show that an increase in the nominal interest rate—which makes it more costly to carry money across periods—reduces the investment of unmatched banks, as banks find it ex-ante optimal to hold less liquidity. An increase in bargaining power θ also lowers $\hat{k}$. Intuitively, when productive banks receive a larger share of the surplus, the benefit of carrying their own funds diminishes, reducing their demand for real money balances. Finally, an increase in matching efficiency η likewise decreases $\hat{k}$, as banks become more likely to access the interbank market and therefore have weaker incentives to bring costly liquidity.

While aggregate investment unambiguously decreases in response to a higher nominal interest rate or greater bargaining power, the effect of matching efficiency is ambiguous. This result reflects two opposing forces. On the one hand, higher matching efficiency increases the probability that banks successfully match, promoting the reallocation of idle liquidity and allowing more banks to attain the unconstrained optimal capital level. On the other hand, it weakens ex-ante incentives to carry liquidity, reducing capital purchased by unmatched banks. Although these two channels work in opposite directions, we show that when the nominal interest rate is close to zero, higher matching efficiency increases aggregate investment.

We also study how the efficiency of the interbank market shapes the sensitivity of aggregate investment to the interest rate. Using a Taylor expansion around $i = 0$, we show that, when the interest rate is very low, an increase in trading probabilities weakens the transmission of changes in the nominal interest rate to investment.

Corollary 1. *For $i \approx 0$, an increase in matching efficiency, η, results in a decrease in $\left|\frac{\partial K}{\partial i}\right|$.*

The following proposition characterizes the comparative statics when the economy is in a scarce-liquidity equilibrium

Proposition 4. *For $i > \bar{i}$, we have*

1. $\frac{\partial k}{\partial i} < 0$, $\frac{\partial \hat{k}}{\partial i} < 0$, $\frac{\partial K}{\partial i} < 0$,
2. $\frac{\partial k}{\partial \theta} < 0$,$\frac{\partial \hat{k}}{\partial \theta} < 0$, $\frac{\partial K}{\partial \theta} < 0$,
3. *there exists a threshold $\bar{\theta}$ such that,*

   i) for $\theta < \bar{\theta}$, $\frac{\partial k}{\partial \eta} > 0$, $\frac{\partial \hat{k}}{\partial \eta} > 0$, $\frac{\partial K}{\partial \eta} > 0$,

   ii) for $\theta > \bar{\theta}$, $\frac{\partial k}{\partial \eta} < 0$, $\frac{\partial \hat{k}}{\partial \eta} < 0$, $\frac{\partial K}{\partial \eta} \gtreqless 0$

In a scarce-liquidity equilibrium, the joint cash holdings of a productive and an unproductive bank are insufficient to reach the unconstrained investment level k^*. As in the abundant-liquidity equilibrium, an increase in either the nominal interest rate or bargaining power reduces the demand for real money balances and lowers the capital purchased by unmatched productive banks. However, unlike in the abundant-liquidity equilibrium, it also reduces investment by matched banks, since even after acquiring the idle liquidity of their counterpart, they remain unable to invest efficiently. In both types of equilibrium, the effect of an increase in either the nominal interest rate or bargaining power on aggregate investment is unambiguously negative.

Changes in bargaining power generate qualitatively similar effects across the two equilibria but operate through an additional channel. As in the abundant-liquidity equilibrium, an increase in bargaining power lowers the demand for real money balances, since it reduces the benefit of holding own funds for a productive bank. In the scarce-liquidity equilibrium, however, there is an extra mechanism at work: when a bank is unproductive but matched, it receives a smaller share of the surplus, which further weakens its incentive to accumulate real balances ex ante.This second effect is absent in the abundant-liquidity equilibrium, because bringing additional money does not increase the surplus from a match; as a result, the marginal value of money reflects only its price.

The impact of a change in matching efficiency, η, on the demand for real money balances—and hence on investment—in the scarce-liquidity equilibrium differs markedly from the abundant-liquidity case. As in the abundant-liquidity equilibrium, higher matching efficiency increases the likelihood that productive banks can improve their liquidity position through the interbank market, thereby weakening their incentives to bring costly liquidity ex-ante. However, in the scarce-liquidity equilibrium a second effect emerges: matched unproductive banks can now lend out all of their idle liquidity and put it to productive use. Consequently, an increase in matching efficiency—by making it easier for these banks to deploy their idle cash profitably—raises their incentive to accumulate liquidity. This channel is absent in the abundant-liquidity equilibrium, where unproductive banks cannot use their marginal liquidity productively.

The relative strength of these two effects—which work in the opposite direction—depends on bargaining power. When the bargaining power of liquidity demanders, θ, is low, unproductive banks retain a larger share of the surplus and therefore benefit more from the increased likelihood of being able to lend their cash. In this case, the second effect dominates, and higher matching efficiency raises the capital of both matched and unmatched banks, thereby increasing aggregate capital. If bargaining power is sufficiently high, however, the additional effect is too weak to offset the reduced incentive to carry liquidity. In this case, an increase in matching efficiency reduces investment by both matched and unmatched banks, but the effect on aggregate capital is ambiguous as banks become more likely to match with a counterpart.

Finally, in contrast to Corollary 1, we show that in a scarce-liquidity equilibrium a more active interbank market strengthens the transmission of monetary policy to aggregate investment.

Corollary 2. *An increase in matching efficiency, η, results in an increase in $\left|\frac{\partial K}{\partial i}\right|$.*

3.6 Optimal Matching Efficiency

Our analysis so far has examined how changes in the nominal interest rate, bargaining power, and matching efficiency affect the investment choices of matched and unmatched banks, as well as aggregate investment. In this section, we turn to the normative implications of the model. In our economy, total welfare depends solely on allocations in the capital market and is given by

$$\frac{1-\beta}{\sigma}\mathcal{W} = \eta\big(f(k) - k\big) + (1-\eta)\big(f(\hat{k}) - \hat{k}\big) \tag{23}$$

From this expression, it follows directly that an increase in either bargaining power or the nominal interest rate always reduces welfare, since both depress capital investment by unmatched banks and, in the scarce-liquidity equilibrium, also by matched banks. By contrast, the welfare effects of changes in matching efficiency are a priori ambiguous. In this section, we therefore ask what level of η maximizes total welfare, and in particular whether full participation ($\eta = \bar{\eta}$), partial participation ($\eta \in (0, \bar{\eta})$), or a full market shutdown ($\eta = 0$) is optimal. It is worth noting that the maximum matching probability for a productive bank is given by $\bar{\eta} = \min\left\{\frac{(1-\sigma)}{\sigma}, 1\right\}$, which is strictly below one whenever $\sigma > \frac{1}{2}$. In this case, there are more productive banks (liquidity demanders) than unproductive banks (liquidity suppliers), so not all liquidity demanders can match with a liquidity supplier even if the number of bilateral meetings is maximized.

The following proposition characterizes the optimal matching efficiency for levels of the nominal interest rate low enough that the economy remains in an abundant-liquidity equilibrium for any admissible value of the matching probability, η.

Proposition 5. *For $i \leq \sigma(1-\theta\bar{\eta})(f'(\frac{k^*}{2}) - 1)$,*

1. *if $\sigma \leq \frac{1}{2}$, optimal matching efficiency is $\eta^* = \bar{\eta}$,*
2. *if $\sigma > \frac{1}{2}$ there exists a threshold $\bar{\theta}$ such that optimal matching efficiency is $\eta^* = \bar{\eta}$ if $\theta < \bar{\theta}$ and $\eta^* = 0$ if $\theta > \bar{\theta}$.*

The first part is straightforward: if there are sufficiently many liquidity suppliers in the interbank market, then under full participation every productive bank is matched and can attain the unconstrained optimal investment level. Full participation therefore achieves the first-best capital allocation. By contrast, if the share of liquidity demanders exceeds that of liquidity suppliers, whether full participation is optimal or whether a full shutdown of the interbank market is preferable depends on the bargaining power of productive banks, θ. Specifically, there exists a threshold value of θ above which shutting down the interbank market becomes optimal. Notably, when the nominal interest rate is sufficiently low, partial participation is never optimal.

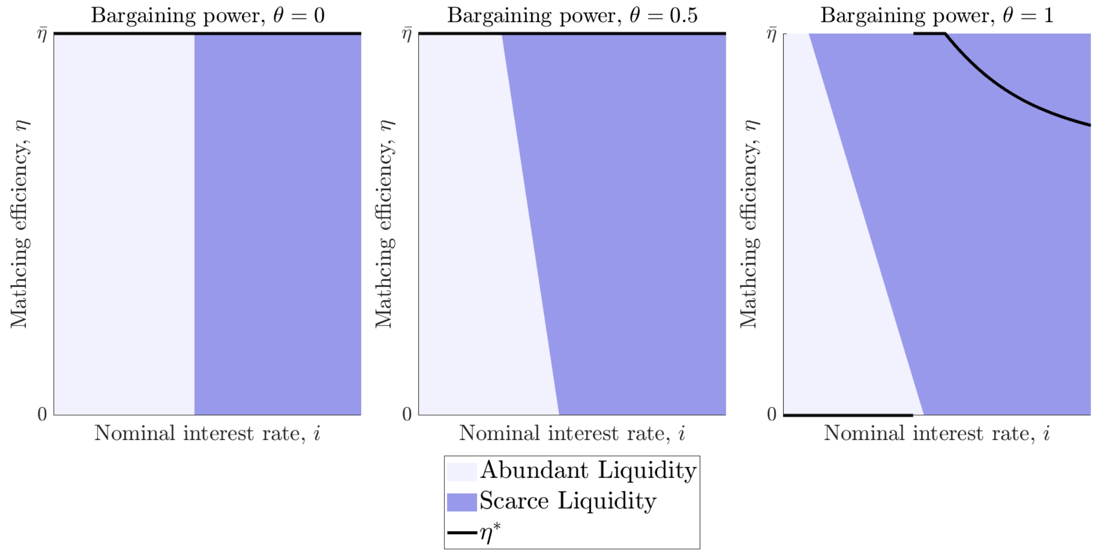
To understand this result, note that an increase in matching efficiency generates two opposing effects. As explained in the previous section, a higher matching rate improves the reallocation of liquidity from unproductive to productive banks, thereby raising investment and aggregate output. At the same time, a higher matching rate reduces banks' incentives to carry liquidity out of the centralized market. This, in turn, lowers the resources available to productive but unmatched banks, reducing their investment. Beyond its impact on aggregate output, higher matching efficiency can either increase or decrease dispersion in the marginal product of capital across banks, thereby generating misallocation and affecting welfare through this channel as well.

When the bargaining power of productive banks is high, unproductive banks—who supply liquidity in bilateral meetings—retain only a small share of the surplus. As a result, they internalize little of the benefit of carrying liquidity. At the same time, high bargaining power incentivizes productive banks to rely more heavily on the liquidity of their unproductive counterparts rather than on their own cash holdings, further weakening incentives to carry liquidity out of the centralized market. Through these channels, when liquidity demanders have high bargaining power, the opportunity to trade money balances dries up liquidity to the point where it may be optimal to fully shut down the market.

In a scarce-liquidity equilibrium, it is more difficult to characterize optimal matching efficiency for all possible parameterizations. However, we are able to show the following result:

Proposition 6. *For $i > \sigma(f'(\frac{1}{2}k^*) - 1)$, if $\theta < 2^{\alpha-1} - \frac{(1-2^{\alpha-1})\sigma}{i}$ then $\frac{\partial \mathcal{W}}{\partial \eta} > 0$ and optimal matching efficiency is $\eta^* = \bar{\eta}$.*

Figure 2: Optimal Matching Efficiency



These figures show the parameter space for which an abundant liquidity equilibrium (white) and scarce liquidity equilibrium (blue) exists and the optimal matching efficiency (black line) for different bargaining powers. This figure is created with $f(k) = k^{0.3}$ and $\sigma = 0.55$. Note that this implies that $\bar{\eta} < 1$.

Proposition 6 states that when bargaining power is sufficiently low, welfare is monotonically increasing in matching efficiency. In this case, higher matching efficiency induces banks to carry more liquidity, which in turn increases investment by both matched and unmatched productive banks. At the same time, an increase in matching efficiency raises the share of productive banks that are successfully matched. Both channels have a positive effect on welfare. Therefore, full participation is optimal, and the optimal matching efficiency is $\eta^* = \bar{\eta}$.

Finally, we provide a sufficient condition under which the optimal participation level is interior, so that neither full participation nor a market shutdown is optimal.

Proposition 7. *There is a threshold $\bar{\theta}(\bar{\eta}, \alpha)$ such that, if $\theta > \bar{\theta}(\bar{\eta}, \alpha)$, there is a nominal interest rate $\tilde{i}$ above which optimal matching efficiency is interior, i.e.* $\eta^* \in$

$(0, \bar{\eta})$.

Proposition 7 establishes the existence of a threshold level for productive banks' bargaining power such that, when bargaining power exceeds this threshold, the optimal matching efficiency becomes interior if the nominal interest rate is sufficiently high. This result does not rely on the assumption that $\sigma > \frac{1}{2}$ (or, equivalently, $\bar{\eta} < 1$). Even when $\bar{\eta} = 1$, interior participation may be optimal. Although full participation equalizes marginal products of capital entirely—since every productive bank is able to find a partner—liquidity is so scarce that increasing matching efficiency beyond a certain point reduces output and makes the economy worse off.

Figure 2 illustrates the optimal matching efficiency for different levels of bargaining power. For $\theta > 0$, the figure displays a region in which increasing matching efficiency beyond a certain threshold induces a switch to a scarce-liquidity equilibrium. In this case, either full participation or a complete market shutdown can be optimal.

3.7 Relation to Previous Studies

We have shown that access to a credit market in which money balances can be reallocated through credit arrangements may reduce aggregate investment and welfare, despite improving the allocation of idle liquidity toward productive agents. Similar results are obtained in Ait Lahcen and Gomis-Porqueras (2021) and Chiu et al. (2018), albeit through different mechanisms. Ait Lahcen and Gomis-Porqueras (2021) study a monetary model in which households differ in their costs of participating in credit markets. In their framework, the reallocation of money balances across households occurs through a competitive banking sector, and heterogeneity in participation costs leads to overconsumption by agents with low participation costs.

Chiu et al. (2018) also consider a competitive banking sector that facilitates liquidity reallocation and show that it may be optimal to limit participation due to a pecuniary externality operating through the price of the decentralized market's special good, which in our case would correspond to the capital good. In our model, this pecuniary externality is absent because capital is produced using a linear technology, implying that its price is fixed and equal to one. Instead, the optimality of limiting market access arises because the reallocation of real balances takes place in a frictional market with search and bargaining. This environment generates a pecuniary externality operating through the price of money, as well as a hold-up problem, since banks internalize the benefits of holding liquidity only to the extent of their share of the surplus. Nevertheless, allowing for a non-linear cost of producing the capital good would introduce a mechanism similar to that in Chiu et al. (2018) and generate additional sources of inefficiency.

Our mechanisms are closer to Berentsen et al. (2014) and Huber and Kim (2019), who study a related environment in which agents face idiosyncratic risk and require a medium of exchange to conduct trades. In their frameworks, agents can sell bonds in either a competitive or a frictional bond market to obtain liquidity after the realization of shocks.[14] By contrast, in our model banks can directly reallocate cash holdings through monetary credit arrangements, whereby productive banks borrow liquidity from unproductive banks in exchange for a promise to repay in the subsequent subperiod.

Similar to our model, the framework in Huber and Kim (2019) features two types of equilibria: an abundant-liquidity equilibrium, in which matched agents are able to attain the unconstrained optimum, and a scarce-liquidity equilibrium, in which both matched and unmatched agents are liquidity constrained and neither can achieve the efficient level of investment. When the economy is in the abundant-liquidity equilibrium, our model and Huber and Kim (2019) display similar positive and normative properties. In particular, an increase in matching efficiency improves welfare when liquidity demanders have low bargaining power, but reduce it otherwise.

In the constrained equilibrium, however, an important difference emerges. In Berentsen et al. (2014) and Huber and Kim (2019), an increase in matching efficiency unambiguously decreases the value of carrying money into the decentralized market. The reason is that, although agents can acquire additional liquidity by trading bonds, the liquidity held by suppliers cannot be reallocated and therefore remains idle. In our framework, this is no longer the case. In the scarce-liquidity equilibrium, the marginal value of money may either increase or decrease as matching efficiency rises, reflecting the presence of two opposing forces. On the one hand, a higher trad-

[14]Berentsen et al. (2014) consider a competitive bond market, whereas Huber and Kim (2019) study a frictional market with search and bargaining.

ing probability tends to reduce money demand because productive banks can access interbank credit more frequently and thus rely on their counterpart's liquidity to finance capital purchases. On the other hand, a higher trading probability tends to increase money demand because unproductive banks can lend out idle balances more often. As discussed earlier, which effect dominates depends on the bargaining power of liquidity demanders.

4 Debt Limit

So far, we have assumed that repayment of monetary loans can be perfectly enforced. In this section, we study how the results change when we introduce a debt limit, $\bar{\Psi}$, which caps the size of interbank loans. This borrowing limit modifies the bargaining problem in the decentralized market and, in turn, affects the value of holding cash for banks. A matched productive bank now faces an additional constraint in the interbank market, as it can borrow at most up to the real debt limit. Formally, the bank's optimization problem becomes

$$(\psi_t, d_t, k_t) \in \arg\max_{\psi_t, d_t, k_t} \left\{ f(k_t) - d_t + \phi_t \psi_t - k_t - f(\hat{k}_t) + \hat{k}_t \right\} \tag{24}$$

subject to

$$f(k_t) - d_t + \phi_t \psi_t - k_t - f(\hat{k}_t) + \hat{k}_t = \frac{\theta}{1-\theta} \left[d_t - \phi_t \psi_t \right] \tag{25}$$

$$\frac{k_t}{\phi_t} \leq m_t^P + \psi_t \tag{26}$$

$$-m_t^P \leq \psi_t \leq m_t^U \tag{27}$$

$$\phi_t \psi_t \leq \bar{\Psi} \tag{28}$$

As in the baseline without a debt limit, the first constraint requires that the productive bank receives a fraction θ of the total surplus. The second constraint states that it cannot spend more cash than it holds, while the third ensures that money can only be reallocated bilaterally within the meeting. The final, new constraint restricts the bank's borrowing in terms of consumption goods, requiring that the value of the loan in real terms cannot exceed $\bar{\Psi}$.

The following lemma characterizes the solution to the bilateral bargaining problem between a productive bank and an unproductive bank, in the presence of an exogenous debt limit.

Lemma 2. *Let (ψ_t, d_t, k_t) be the solution to the bargaining problem between a productive bank, with money holding m_t^P, and an unproductive bank, with money holding m_t^U.*

1. *If $m_t^P + m_t^U \geq \frac{k^*}{\phi_t}$, then*

$$k_t = \begin{cases} k^*, & \text{if } \frac{k^*}{\phi_t} - m_t^P \leq \frac{1}{\phi_t}\bar{\Psi} \\ \phi_t m_t^P + \bar{\Psi}, & \text{otherwise} \end{cases}$$

$$\psi_t \begin{cases} \in \left(\frac{k^*}{\phi_t} - m_t^P, \min\left(m_t^U, \frac{1}{\phi_t}\bar{\Psi}\right)\right), & \text{if } \frac{k^*}{\phi_t} - m_t^P \leq \frac{1}{\phi_t}\bar{\Psi} \\ = \frac{1}{\phi_t}\bar{\Psi}, & \text{otherwise} \end{cases}$$

2. *If $m_t^P + m_t^U < \frac{k^*}{\phi_t}$, then*

$$k_t = \begin{cases} \phi_t(m_t^P + m_t^U), & \text{if } m_t^U \leq \frac{1}{\phi_t}\bar{\Psi} \\ \phi_t m_t^P + \bar{\Psi}, & \text{otherwise} \end{cases}$$

$$\psi_t = \begin{cases} m_t^U & \text{if } m_t^U \leq \frac{1}{\phi_t}\bar{\Psi} \\ \frac{1}{\phi_t}\bar{\Psi}, & \text{otherwise} \end{cases}$$

The interbank loan is

$$d_t = \phi_t\psi_t + (1-\theta)\big[f(k_t) - k_t - f(\hat{k}_t(m_t^P)) + \hat{k}_t(m_t^P)\big]$$

When joint cash holdings $m_t^P + m_t^U$ are sufficient to finance the efficient capital level k^*, the productive bank attains the first-best unless the borrowing constraint binds, in which case the maximum feasible capital level is $\phi_t m_t^P + \bar{\Psi}$. When aggregate liquidity is insufficient to finance k^*, total investment is constrained by available money holdings. If the debt limit does not bind, the unproductive bank transfers all its cash holdings to the productive bank. If instead the debt limit binds, the productive bank borrows from the unproductive bank only up to the limit $\bar{\Psi}$.

The following proposition characterizes the equilibrium

Proposition 8. *For any $i > 0$, there exists a unique monetary equilibrium. If $\bar{\Psi} < \frac{1}{2}k^*$, there exists thresholds, $\bar{i}^1$, $\bar{i}^2$, and $\bar{i}^3$, with $\bar{i}^1 \leq \bar{i}^2 \leq \bar{i}^3$, such that*

1. *If $i \leq \bar{i}^1$, there is an abundant-liquidity equilibrium satisfying*

$$k = k^* \tag{29}$$

$$i = \sigma(1-\theta\eta)\left(f'(\hat{k}) - 1\right) \tag{30}$$

2. *If $\bar{i}^1 < i \leq \bar{i}^2$, there is a debt-constrained equilibrium satisfying*

$$k = \bar{\Psi} + \hat{k} \tag{31}$$

$$i = \sigma(1-\theta\eta)\left(f'(\hat{k}) - 1\right) + \sigma\eta\theta(f'(\bar{\Psi} + \hat{k}) - 1) \tag{32}$$

3. *If $\bar{i}^2 < i \leq \bar{i}^3$, there is a debt-constrained equilibrium satisfying*

$$k = 2\bar{\Psi} \tag{33}$$

$$\hat{k} = \bar{\Psi} \tag{34}$$

4. *If $i > \bar{i}^3$, there is a scarce-liquidity equilibrium satisfying*

$$k = 2\hat{k} \tag{35}$$

$$i = \sigma(1-\theta\eta)\left(f'(\hat{k}) - 1\right) + \sigma\eta\left(f'(2\hat{k}) - 1\right) \tag{36}$$

The thresholds are given by:

$$\bar{i}^1 = \sigma(1-\eta\theta)(f'(k^* - \bar{\Psi}) - 1) \tag{37}$$

$$\bar{i}^2 = \sigma(1-\eta\theta)(f'(\bar{\Psi}) - 1) + \sigma\eta\theta(f'(2\bar{\Psi}) - 1) \tag{38}$$

$$\bar{i}^3 = \sigma(1-\eta\theta)(f'(\bar{\Psi}) - 1) + \sigma\eta(f'(2\bar{\Psi}) - 1) \tag{39}$$

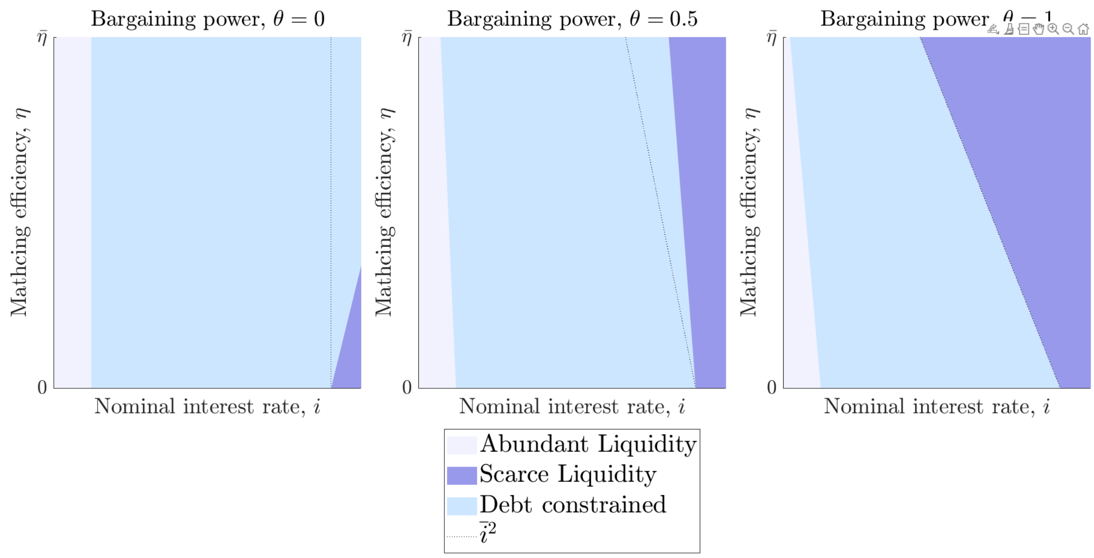


Figure 3: Equilibria Cases

These figures show the parameter space for which an abundant liquidity equilibrium (white), debt constrained equilibrium (blue), and scarce liquidity equilibrium (grey) exists for different bargaining powers. This figure is created with a $\bar{\Psi} = 0.25k^*$, $f(k) = k^{0.3}$ and $\sigma = 0.55$. Note that this implies that $\bar{\eta} < 1$.

As in the case without a debt limit, when the interest rate is sufficiently low ($i < \bar{i}^1$) the economy is in an abundant-liquidity equilibrium, where matched productive banks attain the unconstrained optimal level of capital. Money balances are high enough that banks need to borrow only a small amount from their counterparts to reach the efficient investment level, so the borrowing constraint never binds. Conversely, when the interest rate is sufficiently high ($i > \bar{i}^3$), the economy enters a scarce-liquidity equilibrium in which both matched and unmatched banks invest below the efficient level. In this regime, banks hold so little liquidity that all available funds can be reallocated within a meeting without ever hitting the borrowing limit, and thus the borrowing constraint remains slack here as well. In both of these equilibria, the debt limit is irrelevant for equilibrium allocations, and the comparative statics coincide with those in the baseline model.

Between these two cases, two additional types of equilibria emerge in which the borrowing limit is binding. If $\bar{i}^1 \leq i \leq \bar{i}^2$, the interest rate is still relatively low, so banks carry substantial cash out of the centralized market to finance investment when they are unmatched; in particular, $\hat{k} > \bar{\Psi}$. This means that the liquidity carried by banks is large enough that only part of it can be reallocated in a bilateral meeting without violating the debt limit.[15] Productive banks would like to borrow additional funds from their counterparts to further increase their investment, but the debt limit prevents them from doing so, leaving some of the unproductive banks' liquidity idle.

When the interest rate lies in the interval $\bar{i}^2 \leq i \leq \bar{i}^3$, liquidity becomes relatively scarce, and banks choose to carry only the amount of cash that can be transferred to a productive bank upon matching. In other words, if a bank turns out to be unproductive, the marginal value of holding additional liquidity is zero: any extra cash would remain idle, could not be reallocated because of the debt limit, and would simply impose an inflation tax. In the baseline model without a borrowing limit, banks would optimally carry more liquidity, since all of it could be transferred to a productive bank in a bilateral meeting. With the debt limit in place, however, this is no longer feasible, so banks carry only the minimum amount consistent with the debt constraint. Figure 3 shows the parameter space for which the different equilibria occurs.

In Proposition 9, we characterize comparative statics if the economy is in a debt constrained equilibrium.

Proposition 9. *For $\bar{i}^1 < i < \bar{i}^2$, we have*

1. $\frac{\partial k}{\partial i} < 0$, $\frac{\partial \hat{k}}{\partial i} < 0$, $\frac{\partial K}{\partial i} < 0$,

2. $\frac{\partial k}{\partial \theta} < 0$, $\frac{\partial \hat{k}}{\partial \theta} < 0$, $\frac{\partial K}{\partial \theta} < 0$,

3. $\frac{\partial k}{\partial \eta} < 0$, $\frac{\partial \hat{k}}{\partial \eta} < 0$, $\frac{\partial K}{\partial \eta} \gtreqless 0$.

For $\bar{i}^2 < i < \bar{i}^3$, we have

1. $\frac{\partial k}{\partial i} = 0$, $\frac{\partial \hat{k}}{\partial i} = 0$, $\frac{\partial K}{\partial i} = 0$,

[15] This corresponds to a situation in which the reallocation constraint is slack but the debt limit is binding, so not all idle liquidity can be transferred.

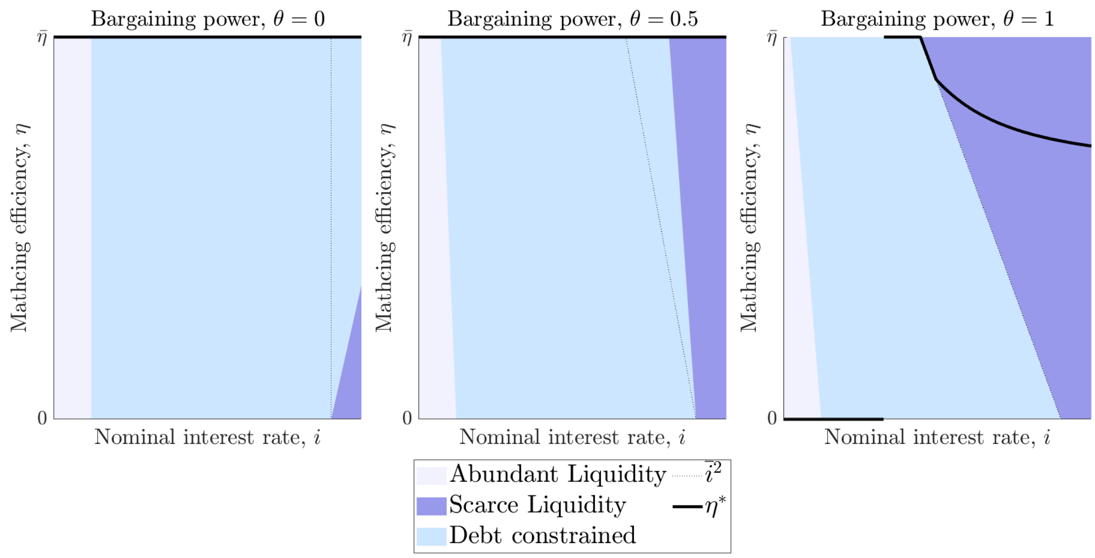


Figure 4: Optimal Matching Efficiency

These figures show the parameter space for which an abundant liquidity equilibrium (white), debt constrained equilibrium (blue), and scarce liquidity equilibrium (grey) exists and the optimal matching efficiency (black line) for different bargaining powers. This figure is created with $\bar{\Psi} = 0.25k^*$, $f(k) = k^{0.3}$ and $\sigma = 0.55$. Note that this implies that $\bar{\eta} < 1$.

2. $\frac{\partial k}{\partial \theta} = 0$, $\frac{\partial \hat{k}}{\partial \theta} = 0$, $\frac{\partial K}{\partial \theta} = 0$,

3. $\frac{\partial k}{\partial \eta} = 0$, $\frac{\partial \hat{k}}{\partial \eta} = 0$, $\frac{\partial K}{\partial \eta} > 0$.

If $\bar{i}^1 < i < \bar{i}^2$, an increase in the nominal interest rate or in bargaining power both have a negative effect on individual and aggregate investment. Higher matching efficiency again has two opposite effects. First, increasing matching efficiency has a negative effect on the amount of liquidity carried. Secondly, increasing matching efficiency has a distributional effect, increasing the share of matched productive banks that can invest more. These two opposite effects imply that the effect on welfare is a priori ambiguous. The direction of the transmission mechanism is also unclear. The return on capital is increasing in the matching probability, while aggregate investment is generally ambiguous. However, for low bargaining power aggregate investment increases. If $\bar{i}^2 < i < \bar{i}^3$, both θ and i have no effect on the allocation, while investment increases with η.

In terms of welfare, Propositions 10 and 11 extend our baseline results to an economy with a debt limit, characterizing how optimal matching efficiency varies across parameter values.[16] Figure 4 illustrates these results by presenting numerical examples of optimal matching efficiency across the different equilibrium regimes. The right panel of the figure, where $\theta = 1$, shows that when bargaining power is high, it may be welfare-improving to limit market efficiency in order to prevent the economy from transitioning from a debt-constrained equilibrium to a scarce-liquidity one.

Proposition 10. *For $i \leq \sigma(1-\bar{\eta}\theta)(f'(k^* - \bar{\Psi}) - 1)$,*

1. *if $\sigma \leq \frac{1}{2}$, optimal matching efficiency is $\eta^* = \bar{\eta}$,*
2. *if $\sigma > \frac{1}{2}$ there exists a threshold $\bar{\theta}$ such that optimal matching efficiency is $\eta^* = \bar{\eta}$ if $\theta < \bar{\theta}$ and $\eta^* = 0$ if $\theta > \bar{\theta}$.*

Proposition 11. *There is a threshold $\bar{\theta}(\bar{\eta}, \alpha)$ such that, if $\theta > \bar{\theta}(\bar{\eta}, \alpha)$, there is a nominal interest rate $\tilde{i}$ above which optimal matching efficiency is interior, i.e. $\eta^* \in (0, \bar{\eta})$.*

5 Conclusion

This paper examines both the positive and normative consequences of interbank trading in a monetary economy where banks must hold liquidity to fund stochastic investment opportunities and subsequently meet bilaterally to reallocate liquidity after these opportunities are realized. Depending on the level of the nominal interest rate, the economy features either abundant- or scarce-liquidity equilibria, each with distinct implications for aggregate investment and the transmission of monetary policy. From a welfare perspective, we show analytically that at low interest rates, the optimal outcome entails either full participation in the interbank market or a complete

[16] Appendix C provides sufficient conditions under which it is optimal to restrict matching efficiency in a debt constrained equilibrium.

market shutdown, depending on the bargaining power of banks with investment opportunities. By contrast, when bargaining power is sufficiently high and interest rates exceed a threshold, welfare is maximized by restricting—but not entirely eliminating—participation in the interbank market.

Our model consolidates banks and entrepreneurs into a single entity, implicitly assuming that banks capture the full surplus from investment opportunities. Introducing this separation and studying how the efficiency of the interbank market affects the transmission to lending rates is a fruitful avenue for future research. Alternatively, incorporating a central bank discount window, as in Williamson (2019), would allow us to study policy-rate pass-through and its dependence on the functioning of the interbank market.

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A Proofs

Proof of Lemma 1. The bargaining problem in a bilateral meeting between a productive bank, holding m_t^P, and an unproductive bank, holding m_t^P is given by Equations 7-10.

First, the lower bound of constraint (Eq. 9) $-m_t^P \leq \psi_t$ will never be binding, given that the production function satisfies Inada conditions. Next, we can re-write the first constraint (Eq. 7) to:

$$d_t = \phi_t \psi_t + (1-\theta)\big[f(k_t) - k_t - f(\hat{k}_t(m_t^P)) + \hat{k}_t(m_t^P)\big]$$

The bargaining problem can then be simplified to

$$\max_{\psi_t, k_t} \theta \Big\{ f(k_t) - k_t - \big[f(\hat{k}_t) - \hat{k}_t\big] \Big\}$$

subject to

$$\frac{k_t}{\phi_t} \leq m_t^P + \psi_t$$
$$\psi_t \leq m_t^U$$

Let λ_1 $(\lambda_2) \geq 0$ denote the multipliers on the first (second) constraint. Then, the first order conditions are given by

$$\theta\,(f'(k_t - 1) = \lambda_1$$
$$\lambda_1 = \lambda_2$$

together with complementary slackness conditions

$$\lambda_1 \left(\frac{k_t}{\phi_t} - m_t^P - \psi_t \right) = 0$$
$$\lambda_2 (m_t^U - \psi_t) = 0$$

If $\phi(m_t^U + m_t^P) < k^*$, then both constraints are both binding, and k_t and ψ_t are given by:

$$k_t = \phi_t (m_t^P + m_t^U)$$
$$\psi_t = m_t^U$$

If $\phi(m_t^P + m_t^U \geq k^*$, then the constraints are not binding, and so k_t and ψ_t are given by:

$$k_t = k^*$$
$$\psi_t = \frac{k^*}{\phi_t} - m_t^P$$

□

Proof of Proposition 1. First, p_t is given by Eq.1, while from Lemma 1 $\{d_t, \psi_t\}$ are given by allocations in the capital market, end-of-period money holdings, and the price of money. The consumption of capital suppliers is given by $c_t^s = k_t^s$. Using market clearing in the capital market $c_t^s = \sigma\eta k_t + \sigma(1-\eta)\hat{k}_t$. The consumption by banks is pinned down by their budget constraints in the CM. These constraints are pinned down by allocations in the capital market, end-of-period money holdings, and the price of money.

From Lemma 1 and Eq. 6 the allocations in the capital market $\{k_t, \hat{k}_t\}$ are determined merely by the real money holdings of banks. Using that the distribution of money holdings is degenerate, $\{k_t, \hat{k}_t\}$ are given by

$$k_t = \begin{cases} k^*, & \text{if } 2\phi m_t \geq k^* \\ \phi 2m_t, & \text{otherwise} \end{cases}$$
$$\hat{k}_t = \begin{cases} k^*, & \text{if } \phi m_t \geq k^* \\ \phi m_t, & \text{otherwise} \end{cases}$$

Next, m_t is given by money market clearing ($m_t = M_t$), the initial money supply (M_0), and the money growth rule μ.Finally, the price of money is then given by Eq. 2, which depends on the expected marginal value of money. From Equations 16, 5, 17, and 4, the value functions of the different banks in the DM are given by:

$$V^P(m_t^P) = \theta\left[f(k_t) - k_t\right] + (1-\theta)\left[f(\hat{k}_t) - \hat{k}_t\right] + \phi_t m_t^P + W(0)$$
$$\hat{V}^P(m_t^P) = f(\hat{k}_t) - \hat{k}_t + \phi_t m_t^P + W(0)$$
$$V^U(m_t^U) = \phi_t m_t^U + (1-\theta)[f(k_t) - \phi_t p_t k_t - f(\hat{k}_t) + \phi_t p_t \hat{k}_t] + W(0)$$
$$\hat{V}^U(m_t^U) = \phi_t m_t^U + W(0)$$

where we distinguish between money holdings of productive and unproductive banks. Taking the derivative of the individual value functions:

$$V_m^P(m_t^P) = \phi_t + \theta[f'(k_t) - 1]\frac{\partial k_t}{\partial m_t^P} + (1-\theta)[f'(\hat{k}_t) - 1]\frac{\partial \hat{k}_t}{\partial m_t^P}$$
$$\hat{V}_m^P(m_t^P) = \phi_t + [f'(\hat{k}_t) - 1]\frac{\partial \hat{k}_t}{\partial m_t^P}$$
$$V_m^U(m_t^U) = \phi_t + (1-\theta)[f'(k_t) - 1]\frac{\partial k_t}{\partial m_t^U}$$
$$\hat{V}_m^U(m_t^U) = \phi_t$$

where we distinguished between the money holdings of productive and unproductive banks and incorporated from Eq. 6 that $\hat{k}_t$ is independent of m_t^U. Combining according to Eq. 3 gives:

$$\mathbb{E}[V'(m)] = \phi + \sigma\eta[f'(k_{t+1}) - 1]\left(\theta\frac{\partial k_t}{\partial m_t^P} + (1-\theta)\frac{\partial k_t}{\partial m_t^U}\right) \\ + \sigma(1-\theta\eta)[f'(\hat{k}_t) - 1]\frac{\partial \hat{k}_t}{\partial m_t^P}$$

Incorporating that money holdings are degenerate across types and substituting in 2 gives:

$$\frac{\phi_t}{\beta} = \phi_{t+1} + \sigma\eta\frac{1}{2}\frac{\partial k_{t+1}}{\partial m_{t+1}}\left(f'(k_{t+1}) - 1\right) + \sigma(1-\theta\eta)\frac{\partial \hat{k}_{t+1}}{\partial m_{t+1}}\left(f'(\hat{k}_{t+1}) - 1\right) \tag{A.1}$$

Therefore, Eq. A.1 gives the path of prices $\{\phi_t\}_{t=0}^{\infty}$. Finally, the consumption market clears due to Walras law.

So far, we assumed that the problem of the banks in the CM is well-defined and entails an interior solution. Using allocations in Lemma 1 and Eq. 6, the expected marginal value of money is:

$$\mathbb{E}[V'(m)] = \begin{cases} \phi, & \text{if } \phi m_t \geq k^* \\ \phi + \sigma(1-\theta\eta)[f'(\hat{k}_t) - 1], & \text{if } \frac{1}{2}k^* \leq \phi m_t < k^* \\ \phi + \sigma\eta(f'(k_t) - 1) + \sigma(1-\theta\eta)[f'(\hat{k}_t) - 1] & \text{if } \phi m_t < \frac{1}{2}k^* \end{cases}$$

Differentiating w.r.t. m gives:

$$\mathbb{E}[V''(m)] = \begin{cases} 0, & \text{if } \phi m_t \geq k^* \\ \sigma(1-\theta\eta)f''(\hat{k}_t), & \text{if } \frac{1}{2}k^* \leq \phi m_t < k^* \\ \sigma\eta 2 f''(k_t) + \sigma(1-\theta\eta)f''(\hat{k}_t) & \text{if } \phi m_t < \frac{1}{2}k^* \end{cases}$$

As $f''(x) < 0$, $V(m)$ is concave in m. Therefore, the problem is well-defined. □

Proof of Proposition 2. From the proof of Proposition 1, the equilibrium is completely pinned down by the price of money given by A.1. From Lemma 1 and Eq. 6, we can consider three different cases 1) $\phi_t \geq \frac{k^*}{M_t}$, 2) $\frac{1}{2}\frac{k^*}{M_t} \leq \phi_t < \frac{k^*}{M_t}$, and 3) $\phi_t \leq \frac{1}{2}\frac{k^*}{M_t}$.

In case 1, we have that $k_t = \hat{k}_t = k^*$. Consequently, Eq. A.1simplifies to

$$\frac{\mu - \beta}{\beta} = i = 0$$

which is inconsistent with an equilibrium as we assume that $\mu > \beta$.

Next, consider case 2. In this case, $k_t = k^*$ and $\hat{k}_t = \phi_t M_t$. Therefore, Eq. A.1 simplifies to

$$\frac{\mu - \beta}{\beta} = i = \sigma(1 - \theta\eta)\left(f'(\phi_t M_t) - 1\right) \tag{A.2}$$

which for a given M_t pins down ϕ_t. Note that in the limit $i \to 0$, that $\hat{k}_t \to k^*$ and the allocation is equivalent to case 1.

In case 3, we have that $k_t = 2\phi_t M_t$ and $\hat{k}_t = \phi_t M_t$. Then Eq. A.1 is given by:

$$\frac{\mu - \beta}{\beta} = i = \sigma(1 - \theta\eta)\left(f'(\phi_t M_t) - 1\right) + \sigma\eta\left(f'(2\phi_t M_t) - 1\right) \tag{A.3}$$

Case 2 occurs as long as $\phi_t M_t \geq \frac{1}{2}k^*$. As the RHS of Eq. A.2 is decreasing in $\phi_t M_t$ or $\hat{k}_t$, the inequality holds as long as:

$$i \leq \bar{i} = \sigma(1 - \theta\eta)\left(f'\left(\frac{k^*}{2}\right) - 1\right)$$

Using A.3 to check that $\phi_t M_t < \frac{1}{2}k^*$ gives the condition that $i > \bar{i}$. □

Proof of Proposition 3. If $i \leq \bar{i}$, the allocation is given by Equations 18 and 19. It is clear that $k = k^*$ is independent of θ, η and i. Implicitly differentiating Eq. 19 gives:

$$\frac{\partial \hat{k}}{\partial i} = \frac{1}{\sigma(1 - \theta\eta)f''(\hat{k})} \tag{A.4}$$

$$\frac{\partial \hat{k}}{\partial \theta} = \frac{\eta\left(f'(\hat{k}) - 1\right)}{(1 - \theta\eta)f''(\hat{k})} \tag{A.5}$$

$$\frac{\partial \hat{k}}{\partial \eta} = \frac{\theta\left(f'(\hat{k}) - 1\right)}{(1 - \theta\eta)f''(\hat{k})} \tag{A.6}$$

as $f''(k) < 0$ $\hat{k}$ is decreasing in the nominal interest rate, bargaining power and matching efficiency. Aggregate capital invested is $K = \sigma\eta k^* + \sigma(1-\eta)\hat{k}$. Therefore, we have that:

$$\begin{aligned}
\frac{\partial K}{\partial i} &= \sigma(1-\eta)\frac{\partial \hat{k}}{\partial i} < 0 \\
\frac{\partial K}{\partial \theta} &= \sigma(1-\eta)\frac{\partial \hat{k}}{\partial \theta} < 0 \\
\frac{\partial K}{\partial \eta} &= \sigma(k^* - \hat{k}) + \sigma(1-\eta)\frac{\partial \hat{k}}{\partial \eta}
\end{aligned}$$

If $\theta = 0$, it is clear that $\frac{\partial K}{\partial \eta}$ is increasing. If $\theta > 0$, we start by taking the derivative w.r.t. i gives:

$$\frac{\partial^2 K}{\partial \eta \partial i} = \sigma \left(-1 + \frac{(1-\eta)\theta}{(1-\theta\eta)} \frac{(f''(\hat{k})^2 - f'''(\hat{k})(f'(\hat{k})-1)}{f''(\hat{k})^2} \right) \frac{\partial \hat{k}}{\partial i}$$

At $i = 0$, $\hat{k} = k^*$ independent of η and θ. Then $\left.\frac{\partial K}{\partial \eta}\right|_{i=0} = 0$ and $\left.\frac{\partial^2 K}{\partial \eta \partial i}\right|_{i=0}$ is:

$$\left.\frac{\partial^2 K}{\partial \eta \partial i}\right|_{i=0} = -\sigma \left(\frac{1-\theta}{(1-\theta\eta)} \right) \left.\frac{\partial \hat{k}}{\partial i}\right|_{i=0}$$

As $\frac{\partial \hat{k}}{\partial i} < 0$, this expression is positive. Therefore, for $i \approx 0$, K increases in η.

□

Proof of Corollary 1. A first-order Taylor expansion of real money holdings (Eq. 19) around k^* yields

$$\hat{k} - k^* \approx \frac{i}{f''(k^*)\sigma(1-\eta\theta)}$$

Similarly, a first-order expansion around aggregate capital gives:

$$\begin{aligned}
K &= \sigma\eta k^* + \sigma(1-\eta)\hat{k} \\
&\approx \sigma k^* + \frac{(1-\eta)i}{f''(k^*)(1-\eta\theta)}
\end{aligned}$$

Taking the derivatives w.r.t. i give:

$$\frac{\partial K}{\partial i} = \frac{(1-\eta)}{(1-\eta\theta)f''(k^*)}$$

As $\frac{(1-\eta)}{(1-\eta\theta)}$ is decreasing in η, we find that $\frac{\partial}{\partial \eta}\left|\frac{\partial K}{\partial i}\right| < 0$. □

Proof of Proposition 4. If $i > \bar{i}$, the allocation is given by Eq. 20 and Eq. 21. As $k = 2\hat{k}$, it is sufficient to analyze $\hat{k}$. Using the implicit function theorem on Eq. 21:

$$\frac{\partial \hat{k}}{\partial i} = \frac{1}{\sigma(1-\theta\eta)f''(\hat{k}) + 2\sigma\eta f''(2\hat{k})} \tag{A.7}$$

$$\frac{\partial \hat{k}}{\partial \theta} = \frac{\eta\left(f'(\hat{k})-1\right)}{(1-\theta\eta)f''(\hat{k}) + 2\eta f''(2\hat{k})} \tag{A.8}$$

$$\frac{\partial \hat{k}}{\partial \eta} = -\frac{f'(2\hat{k}) - 1 - \theta(f'(\hat{k})-1)}{\eta 2 f''(2\hat{k}) + (1-\eta\theta)f''(\hat{k})} \tag{A.9}$$

As $f''(k) < 0$, the denominator is strictly negative. Therefore, the first two equations are strictly decreasing, while for the last equation it depends on θ.

Using the properties of the homogeneous function, we can re-write Eq. 21 to:

$$f'(\hat{k}) = \frac{i + \sigma(\eta(1-\theta)+1)}{\sigma(\eta(2^{\alpha-1}-\theta)+1)}$$

Differentiating the RHS w.r.t. η gives:

$$\frac{\sigma\left[(1-2^{\alpha-1})\sigma - (2^{\alpha-1}-\theta)i\right]}{[\sigma(\eta(2^{\alpha-1}-\theta)+1)]^2}$$

The denominator is strictly positive, hence the RHS is increasing in η if:

$$\theta > 2^{\alpha-1} - \frac{(1-2^{\alpha-1})\sigma}{i} \tag{A.10}$$

As the LHS is decreasing in $\hat{k}$, we find a threshold in θ above (below) $\hat{k}$ is decreasing (increasing) in matching efficiency.

Aggregate capital invested is $K = \sigma\eta 2\hat{k} + \sigma(1-\eta)\hat{k}$. Therefore, we have that

$$\frac{\partial K}{\partial i} = \sigma(1+\eta)\frac{\partial \hat{k}}{\partial i} < 0 \tag{A.11}$$

$$\frac{\partial K}{\partial \theta} = \sigma(1+\eta)\frac{\partial \hat{k}}{\partial \theta} < 0 \tag{A.12}$$

$$\frac{\partial K}{\partial \eta} = \sigma(k^* - \hat{k}) + \sigma(1+\eta)\frac{\partial \hat{k}}{\partial \eta} \tag{A.13}$$

If $\theta < \bar{\theta}$, $\frac{\partial K}{\partial \eta} > 0$, while if $\theta > \bar{\theta}$ the sign is ambiguous.

□

Proof of Corollary 2. Combining Equations A.11 and A.7 gives:

$$\frac{\partial K}{\partial i} = \frac{(1+\eta)}{f''(\hat{k})\left((1-\eta\theta) + 2^{\alpha-1}\eta\right)} < 0$$

which is strictly negative. Differentiating the absolute value w.r.t. to η gives:

$$\frac{\partial}{\partial \eta}\left|\frac{\partial K}{\partial i}\right| = -\frac{1}{f''(\hat{k})}\left[\frac{(1-2^{\alpha-1}+\theta)}{((1-\eta\theta)+2^{\alpha-1}\eta)^2}\right] - \frac{f'''(\hat{k})}{f''(\hat{k})^3}\left(\frac{(1+\eta)}{((1-\eta\theta)+2^{\alpha-1}\eta)^2}\right) > 0$$

where we used Eq. A.9. □

Proof of Proposition 5. From Eq 37, $\bar{i}$ is a decreasing function in η. Therefore, if $i \leq \sigma(1-\bar{\eta}\theta)(f'(\frac{k^*}{2}) - 1)$, the equilibrium is abundant for all $\eta \in [0, \bar{\eta}]$.

Differentiating welfare given by Eq. 23 w.r.t. η gives:

$$\begin{aligned}\frac{(1-\beta)}{\sigma}\frac{\partial \mathcal{W}}{\partial \eta} &= f(k^*) - k^* - f(\hat{k}) + \hat{k} + (1-\eta)\frac{\theta(f'(\hat{k})-1)^2}{(1-\theta\eta)f''(\hat{k})} \\ &= f(k^*) - k^* - f(\hat{k}) + \hat{k} + (1-\eta)\frac{\theta(\alpha f(\hat{k}) - \hat{k})^2}{(1-\theta\eta)\alpha(\alpha-1)f(\hat{k})}\end{aligned}$$

where we used Eq.A.6 and the properties of a homogeneous function. Similarly, the second derivative is given by

$$\frac{(1-\beta)}{\sigma}\frac{\partial^2\mathcal{W}}{\partial\eta\partial\eta}=\frac{\theta(\alpha f(\hat{k})-\hat{k})^2}{(1-\theta\eta)\alpha(1-\alpha)f(\hat{k})} - \left((\alpha f'(\hat{k})-1)+\frac{(1-\eta)\theta}{(1-\theta\eta)}\alpha(1-\alpha))\frac{(\alpha f(\hat{k})-\hat{k})^2(2-\alpha)}{f(\hat{k})\hat{k}}\right)\frac{\partial\hat{k}}{\partial\eta}$$

As the terms in brackets is strictly positive and from Eq. A.6 $\frac{\partial\hat{k}}{\partial\eta}<0$, expected welfare is convex in η. Consequently, we only have to compare welfare at $\eta=0$ and at the maximum $\eta=\bar{\eta}$.

Denote with $\tilde{k}$, capital invested at $\eta=0$ and $\hat{k}$ capital of unmatched banks at $\eta=\bar{\eta}$. Welfare at $\eta=0$ and $\eta=\bar{\eta}$ is given by:

$$\frac{(1-\beta)}{\sigma}\mathcal{W}\bigg|_{\eta=0}=f(\tilde{k})-\tilde{k}$$
$$\frac{(1-\beta)}{\sigma}\mathcal{W}\bigg|_{\eta=\bar{\eta}}=\bar{\eta}(f(k^*)-k^*)+(1-\bar{\eta})(f(\hat{k})-\hat{k})$$

It is obvious that if $\bar{\eta}=1$ which requires $\sigma\leq\frac{1}{2}$, the optimal matching probability is $\eta^*=\bar{\eta}$ as $\frac{(1-\beta)}{\sigma}\mathcal{W}\Big|_{\eta=\bar{\eta}}>\frac{(1-\beta)}{\sigma}\mathcal{W}\Big|_{\eta=0}$. If $\sigma>\frac{1}{2}$, then we look at $\hat{k}$ and $\tilde{k}$:

$$i=\sigma(f'(\tilde{k})-1)$$
$$i=\sigma(1-\bar{\eta}\theta)(f'(\hat{k})-1)$$

As $f''(k)<0$, $\tilde{k}>\hat{k}$ for $\theta>0$ while with $\theta=0$ they coincide. Therefore, if $\theta=0$

$\eta^* = \bar{\eta}$. Re-writing these equations and dividing by $f'(k^*) = 1$ allows us to find:

$$
\begin{aligned}
\frac{f'(\tilde{k})}{f'(k^*)} &= \tilde{\rho}^{\alpha-1} = \frac{i+\sigma}{\sigma} \\
\tilde{\rho} &= \left[\frac{i+\sigma}{\sigma}\right]^{\frac{1}{\alpha-1}} \\
\frac{f'(\hat{k})}{f'(k^*)} &= \hat{\rho}^{\alpha-1} = \frac{i+\sigma(1-\theta\bar{\eta})}{\sigma(1-\theta\bar{\eta})} \\
\hat{\rho} &= \left[\frac{i+\sigma(1-\theta\bar{\eta})}{\sigma(1-\theta\bar{\eta})}\right]^{\frac{1}{\alpha-1}}
\end{aligned}
$$

where $\tilde{\rho}$ ($\hat{\rho}$) solves $\tilde{\rho}\tilde{k} = k^*$ ($\hat{\rho}\hat{k} = k^*$) with $\hat{\rho} \in (1, 0.5]$, as at $\bar{i}$ $\hat{k} = \frac{1}{2}k^*$, and $\tilde{\rho} \geq \hat{\rho}$. This allows us to write welfare as:

$$
\begin{aligned}
\frac{(1-\beta)}{\sigma}\mathcal{W}\bigg|_{\eta=0} &= \tilde{\rho}^{\alpha} f(k^*) - \tilde{\rho}k^* \\
\frac{(1-\beta)}{\sigma}\mathcal{W}\bigg|_{\eta=\bar{\eta}} &= \bar{\eta}(f(k^*) - k^*) + (1-\bar{\eta})(\hat{\rho}^{\alpha} f(k^*) - \hat{\rho}k^*)
\end{aligned}
$$

Next, we check when full participation is better than a complete shutdown:

$$
\begin{aligned}
\frac{(1-\beta)}{\sigma}\mathcal{W}\bigg|_{\eta=\bar{\eta}} &> \frac{(1-\beta)}{\sigma}\mathcal{W}\bigg|_{\eta=0} \\
(\bar{\eta} + (1-\bar{\eta})\hat{\rho}^{\alpha} - \tilde{\rho}^{\alpha})f(k^*) - (\bar{\eta} + (1-\bar{\eta})\hat{\rho} - \tilde{\rho})k^* &> 0 \\
(\bar{\eta} + (1-\bar{\eta})\hat{\rho}^{\alpha} - \tilde{\rho}^{\alpha} - \alpha(\bar{\eta} + (1-\bar{\eta})\hat{\rho} - \tilde{\rho})f(k^*) &> 0 \\
\Omega f(k^*) &> 0
\end{aligned}
$$

where we used that $f'(k^*) = 1$ or $\alpha f(k^*) = k^*$ and $\Omega = (\bar{\eta} + (1-\bar{\eta})\hat{\rho}^{\alpha} - \tilde{\rho}^{\alpha} - \alpha(\bar{\eta} + (1-\bar{\eta})\hat{\rho} - \tilde{\rho})$. Therefore, we have to check when $\Omega > 0$.

First, note that at $\theta = 0$ that $\Omega(0) = \bar{\eta}(1 - \alpha - \hat{\rho}^{\alpha} + \alpha\hat{\rho}) > 0$. At $\theta = 1$, $\Omega(1)$ is given by:

$$\Omega(1) = \bar{\eta}(1-\alpha) + (1-\bar{\eta})\left(\left[\frac{i+\sigma(1-\bar{\eta})}{\sigma(1-\bar{\eta})}\right]^{\frac{\alpha}{\alpha-1}} - \alpha\left[\frac{i+\sigma(1-\bar{\eta})}{\sigma(1-\bar{\eta})}\right]^{\frac{1}{\alpha-1}}\right)$$
$$-\left(\left[\frac{i+\sigma}{\sigma}\right]^{\frac{\alpha}{\alpha-1}} - \alpha\left[\frac{i+\sigma}{\sigma}\right]^{\frac{1}{\alpha-1}}\right)$$

If $i \to 0$, then $\Omega(1) = 0$. Differentiating $\Omega(1)$ w.r.t. i gives:

$$\left.\frac{\partial \Omega}{\partial i}\right|_{\theta=1} = \frac{\alpha}{\sigma(\alpha-1)}\Big(\hat{\rho}(1-\hat{\rho}^{1-\alpha}) - \tilde{\rho}(1-\tilde{\rho}^{1-\alpha})\Big)$$

Define $h(\rho) = \rho(1-\rho)^{1-\alpha}$, this function is decreasing in ρ for $\rho \in [0.5, 1)$. Therefore, as at $\theta = 1$ $\tilde{\rho} > \hat{\rho}$ the term in bracket is positive. However, as $\alpha < 1$ we find that $\frac{\partial \Omega}{\partial i}\big|_{\theta=1} < 0$ and $\Omega(1) < 0$ for $i > 0$.

Finally, differentiating Ω with respect to θ gives:

$$\frac{\partial \Omega}{\partial \theta} = (1-\bar{\eta})\alpha(\hat{\rho}^{\alpha-1} - 1)\frac{\partial \hat{\rho}}{\partial \theta}$$

as $\hat{\rho}^{\alpha-1} - 1 > 0$ for $\rho \in [0.5, 1)$ and $\alpha \in (0, 1)$ and $\frac{\partial \hat{\rho}}{\partial \theta} < 0$, we find that Ω is decreasing in θ. As $\Omega(0) > 0 > \Omega(1)$, there is a cut-off point in θ below which $\eta^* = \bar{\eta}$ and above $\eta^* = 0$. □

Proof of Proposition 6. From Eq.37, $\bar{i}$ is a decreasing function in η. Consequently, for $i \geq \sigma(f'(\frac{1}{2}k^*) - 1)$, the allocation is always given by the scarce equilibrium case. Next, differentiating welfare (Eq. 23) with respect to η gives:

$$\frac{(1-\beta)}{\sigma}\frac{\partial \mathcal{W}}{\partial \eta} = \Big(f(2\hat{k}) - 2\hat{k} - f(\hat{k}) + \hat{k}\Big) + \Big(2\eta(f'(2\hat{k}) - 1) + (1-\eta)(f'(\hat{k}) - 1)\Big)\frac{\partial \hat{k}}{\partial \eta}$$

As $f(k) - k$ is increasing for $i > 0$, the first term is weakly positive. Similarly, the second term is strictly positive if $\frac{\partial \hat{k}}{\partial \eta} > 0$. From the proof of Proposition 3, $\frac{\partial \hat{k}}{\partial \eta} > 0$ if $\theta < \bar{\theta}$, where $\bar{\theta}$ is given by Eq. A.10. □

Proof of Proposition 7. From Eq. 37, $\bar{i}$ is a decreasing function in η. Consequently, for $i \geq \sigma(f'(\frac{1}{2}k^*) - 1)$, the allocation is always given by the scarce equilibrium case. Next, differentiating welfare (Eq. 23) with respect to η gives:

$$\frac{(1-\beta)}{\sigma}\frac{\partial \mathcal{W}}{\partial \eta} = f(2\hat{k}) - 2\hat{k} - f(\hat{k}) + \hat{k} + \left(2\eta(f'(2\hat{k}) - 1) + (1-\eta)(f'(\hat{k}) - 1)\right)\frac{\partial \hat{k}}{\partial \eta}$$

Using Eq. A.9 and the properties of a homogeneous function, we can re-write this to:

$$\begin{aligned}\frac{(1-\beta)}{\sigma}\frac{\partial \mathcal{W}}{\partial \eta} &= f(\hat{k})\left[\frac{(1-\alpha)(1-\eta\theta+\eta 2^{\alpha-1})(2^{\alpha}-1) + \alpha(\eta 2^{\alpha} + 1 - \eta)(2^{\alpha-1}-\theta)}{(1-\alpha)(1-\eta\theta+\eta 2^{\alpha-1})}\right] \\ &- \hat{k}\left[\frac{(1-\alpha)(1-\eta\theta+\eta 2^{\alpha-1}) + (1+\eta)(2^{\alpha-1}-\theta) + (1-\theta)(\eta 2^{\alpha}+1-\eta)}{(1-\alpha)(1-\eta\theta+\eta 2^{\alpha-1})}\right] \\ &+ \frac{\hat{k}^2}{\alpha f(\hat{k})}\frac{(1+\eta)(1-\theta)}{(1-\alpha)(1-\eta\theta+\eta 2^{\alpha-1})}\end{aligned}$$

Evaluate at $\eta = 0$:

$$\begin{aligned}\frac{(1-\beta)}{\sigma}\frac{\partial \mathcal{W}}{\partial \eta}\bigg|_{\eta=0} &= f(\hat{k})\left[\frac{(1-\alpha)(2^{\alpha}-1) + \alpha(2^{\alpha-1}-\theta)}{(1-\alpha)}\right] \\ &- \hat{k}\left[\frac{(1-\alpha) + (2^{\alpha-1}-\theta)}{(1-\alpha)}\right] \\ &+ \frac{\hat{k}^2}{\alpha f(\hat{k})}\frac{(1-\theta)}{(1-\alpha)}.\end{aligned}$$

Note that at $\hat{k} = 0$, the expression is equal to zero. Differentiating w.r.t. $\hat{k}$ and evaluating at $\hat{k} \to 0$ gives:

$$\begin{aligned}\lim_{\hat{k}\to 0}\frac{(1-\beta)}{\sigma}\frac{\partial^2 \mathcal{W}}{\partial \eta \partial k}\bigg|_{\eta=0} &= \left(\lim_{\hat{k}\to 0} f'(k)\left[\frac{(1-\alpha)(2^{\alpha}-1) + \alpha(2^{\alpha-1}-\theta)}{(1-\alpha)}\right]\right. \\ &- \left[\frac{(1-\alpha) + (2^{\alpha-1}-\theta)}{(1-\alpha)}\right]\end{aligned}$$

As $\lim_{k\to 0} f'(k) = \infty$ and that $(1-\alpha)(2^{\alpha}-1) + \alpha(2^{\alpha-1}-\theta) > 0$, the term in brackets is positive. Therefore, for small $\hat{k}$ $\frac{(1-\beta)}{\sigma}\frac{\partial \mathcal{W}}{\partial \eta}\Big|_{\eta=0} > 0$. Note that as $\hat{k}$ is decreasing in i independent of α, θ, and η. Then for high nominal interest rates, we have that $\frac{(1-\beta)}{\sigma}\frac{\partial \mathcal{W}}{\partial \eta}\Big|_{\eta=0} > 0$.

Next, evaluate $\frac{(1-\beta)}{\sigma}\frac{\partial \mathcal{W}}{\partial \eta}$ at $\eta = \bar{\eta}$:

$$\begin{aligned}\left.\frac{(1-\beta)}{\sigma}\frac{\partial \mathcal{W}}{\partial \eta}\right|_{\eta=\bar{\eta}} &= f(\hat{k})\left[\frac{(1-\alpha)(1-\bar{\eta}\theta+\bar{\eta}2^{\alpha-1})(2^{\alpha}-1)+\alpha(\bar{\eta}2^{\alpha}+1-\bar{\eta})(2^{\alpha-1}-\theta)}{(1-\alpha)(1-\bar{\eta}\theta+\bar{\eta}2^{\alpha-1})}\right] \\ &- k\left[\frac{(1-\alpha)(1-\bar{\eta}\theta+\bar{\eta}2^{\alpha-1})+(1+\bar{\eta})(2^{\alpha-1}-\theta)+(1-\theta)(\bar{\eta}2^{\alpha}+1-\bar{\eta})}{(1-\alpha)(1-\bar{\eta}\theta+\bar{\eta}2^{\alpha-1})}\right] \\ &+ \frac{k^2}{\alpha f(k)}\frac{(1+\bar{\eta})(1-\theta)}{(1-\alpha)(1-\bar{\eta}\theta+\bar{\eta}2^{\alpha-1})}.\end{aligned}$$

Evaluating again the derivative w.r.t. $\hat{k}$ at the limit of $\hat{k} \to 0$. A sufficient condition for $\left.\frac{(1-\beta)}{\sigma}\frac{\partial \mathcal{W}}{\partial \eta}\right|_{\eta=\bar{\eta}} < 0$ for high i is:

$$(1-\alpha)(1-\bar{\eta}\theta+\bar{\eta}2^{\alpha-1})(2^{\alpha}-1)+\alpha(\bar{\eta}2^{\alpha}+1-\bar{\eta})(2^{\alpha-1}-\theta) < 0$$

as the denominator is strictly positive. Re-writing gives:

$$\theta > \frac{(1-\alpha)(1+\bar{\eta}2^{\alpha-1})(2^{\alpha}-1)+\alpha 2^{\alpha-1}(\bar{\eta}2^{\alpha}+1-\bar{\eta})}{(1-\alpha)\bar{\eta}(2^{\alpha}-1)+\alpha(\bar{\eta}2^{\alpha}+1-\bar{\eta})} \equiv \bar{\theta}(\alpha,\bar{\eta})$$

Therefore, for a nominal interest rate such that we are in the scarce liquidity, there is a nominal interest above which $\left.\frac{(1-\beta)}{\sigma}\frac{\partial \mathcal{W}}{\partial \eta}\right|_{\eta=\bar{\eta}} < 0$, and $\left.\frac{(1-\beta)}{\sigma}\frac{\partial \mathcal{W}}{\partial \eta}\right|_{\eta=0} > 0$. □

Proof of Lemma 2. The bargaining problem is given by Equations 24-28. The constraint $-m_t^P \leq \psi_t$ will never be binding, given that the production function satisfies Inada conditions. Secondly, we can re-write the third and second constraint (Eq. 27 and Eq. 28) as $\phi_t\psi_t \leq \min[\bar{\Psi}, \phi_t m_t^U]$. Re-writing the first constraint to:

$$d_t = \phi_t\psi_t + (1-\theta)\big[f(k_t) - k_t - f(\hat{k}_t(m_t^P)) + \hat{k}_t(m_t^P)\big]$$

We can re-write the problem as:

$$\max_{\psi_t, k_t} \theta\Big\{f(k_t) - k_t - \big[f(\hat{k}_t) - \hat{k}_t\big]\Big\}$$

subject to

$$\frac{k_t}{\phi_t} \leq m_t^P + \psi_t$$
$$\phi_t \psi_t \leq \min[\bar{\Psi}, \phi_t m_t^U]$$

Let λ_1 $(\lambda_2) \geq 0$ denote the multipliers on the first (second) constraint. Then, the first order conditions are given by

$$\theta\,(f'(k_t - 1) = \lambda_1$$
$$\lambda_1 = \lambda_2$$

together with complementary slackness conditions

$$\lambda_1 \left(\frac{k_t}{\phi_t} - m_t^P - \psi_t \right) = 0$$
$$\lambda_2 (m_t^U - \psi_t) = 0$$

There are then three potential cases. First, if $k^* - \phi_t m_t^P \leq \min[\bar{\Psi}, \phi m_t^U]$, then k_t, ψ_t are given by:

$$k_t = k^*$$
$$\psi_t \in \left(\frac{k^*}{\phi} - m_t^P, \min\left[\frac{\bar{\Psi}}{\phi}, m_t^U \right] \right)$$

Next, if $k^* - \phi_t m_t^P \leq \min[\bar{\Psi}, \phi m_t^U]$, then k_t, ψ_t are given by:

$$k_t = \phi_t m_t^P + \min[\bar{\Psi}, \phi m_t^U]$$
$$\psi_t = \min[\bar{\Psi}, \phi m_t^U]$$

□

Proof of Proposition 8. Before analysing equilibria, it is useful to check the maximum loan size in the abundant and slack liquidity equilibrium. From Equations 14 and 12, the loan size in an abundant and scarce equilibrium are respectively given by:

$$\psi = \begin{cases} p_t k^* - m_t, & \text{if } i \leq \bar{i} \\ m_t, & \text{if } i > \bar{i} \end{cases}$$

where we incorporated that money holdings are degenerate across banks and assume that a matched productive bank prefers to use it's own money holdings before borrowing. This assumption is WLOG as it minimizes the loan size of a bank. As $\hat{k} = m_t$, if $i < \bar{i}$, ψ is increasing in i from Eq. A.4, whereas if $i > \bar{i}$ ψ is decreasing in i from Eq. A.7. The maximum loan size is given then by $\phi\psi = \frac{1}{2}k^*$ and corresponds with $\bar{i}$. Consequently, only if $\bar{\Psi} < \frac{1}{2}k^*$ the debt limit might bind.

From the terms-of-trade in Lemma 2, there are three different cases. First, if all constraints are slack we are in the abundant equilibrium described in Proposition 2. Secondly, if the reallocation constraint (Eq. 27) binds and the debt limit (Eq. 28) is slack we are in the scarce liquidity equilibrium described in Proposition 2. In other cases, the debt limit binds. We can distinguish between two different cases: i) the debt limit binds and the reallocation constraint is slack, ii) both the debt limit and the reallocation constraint binds.

In the first case, $k_t = \phi m_t^P + \bar{\Psi}$, which is independent of m^U. Therefore, we find that Eq. A.1) is:

$$i = \sigma(1 - \theta\eta)\left(f'(\hat{k}) - 1\right) + \sigma\eta\theta(f'(\hat{k} + \bar{\Psi}) - 1)$$

which pins down $\hat{k}$ and by extension ϕm.

In case 2, from Equations 27 and 28 it is clear that k is non-differentiable in m_t^U. Let $k(m^U) = f(m^U)$ denote k as a function of m^U and $A = \frac{1}{\phi}\bar{\Psi}$, the left and right-hand derivatives are respectively:

$$f'_{-}(A) = \lim_{m_t^U \to A^-} \frac{f(A+h) - f(A)}{h} = \phi f'(A^-)$$
$$f'_{+}(A) = \lim_{m_t^U \to A^+} \frac{f(A+h) - f(A)}{h} = 0$$

This implies that at $m = \frac{1}{\phi_t}\bar{\Psi}$, the DM value function is non-differentiable. In this case, the money holdings is pinned down by the binding constraints $\hat{k} = \phi m = \bar{\Psi}$ and $k = 2\bar{\Psi}$.

To check, when each case occurs we inspect the marginal value of money. The marginal value of money for a bank is given by:

$$V_m(m) = \begin{cases} \phi, & \text{if } m \geq m^* \\ \phi + \phi\sigma(1-\eta\theta)(f'(\phi m) - 1), & \text{if } m^* < m \leq p(k^* - \bar{\Psi}) \\ \phi + \phi\sigma(1-\eta\theta)(f'(\phi m) - 1) + \sigma\eta\theta(f'(\phi m + \bar{\Psi}) - 1), & \text{if } p(k^* - \bar{\Psi}) < m < p\bar{\Psi} \\ \phi + \phi\sigma(1-\eta\theta)(f'(\phi m) - 1) + \sigma\eta(f'(\phi m + \bar{\Psi}) - 1), & \text{if } m < p\bar{\Psi} \end{cases}$$

where we used that $m^* = pk^*$. Note that at the other thresholds the derivatives are continuous and that $V(m)$ is weakly concave except at $\phi m = \bar{\Psi}$. This implies that $V_m(m)$ is a continuous function with kinks at the boundaries and undefined at $\phi m = \bar{\Psi}$. In addition, for i) $\phi m^* < \phi m < k^* - \bar{\Psi}$, ii) $k^* - \bar{\Psi} < \phi m < \bar{\Psi}$, and $\phi m < \bar{\Psi}$, the marginal value of money is decreasing in m.

From 2, the optimal choice of money holdings is given by:

$$-\frac{\phi}{\beta} + V_m(m_{+1}) = 0$$

If at $\phi m = \bar{\Psi}$, the left-derivative is increasing and the right-derivative is decreasing, $\phi m = \Psi$ might consists of a local optimum. However, as the problem is weakly concave this is also our global optimum. Using that $\frac{\phi}{\phi_{+1}\beta} - 1 = i$ evaluating at the left-hand derivative and right-hand derivative gives two cut-off point in the nominal interest rate:

$$i^2 = \sigma(1-\eta\theta)(f'(\bar{\Psi})-1) + \sigma\eta\theta(f'(2\bar{\Psi})-1)$$
$$i^3 = \sigma(1-\eta\theta)(f'(\bar{\Psi})-1) + \sigma\eta(f'(2\bar{\Psi})-1)$$

Therefore, for $i^2 \leq i \leq i^3$, $\phi m = \bar{\Psi}$. Using the other threshold, we find that the optimal money holdings are given by

$$\begin{cases} i = \sigma(1-\eta\theta)(f'(\phi m)-1), & \text{if } i \leq i^1 \\ i = \sigma(1-\eta\theta)(f'(\phi m)-1) + \sigma\eta\theta(f'(\phi m + \bar{\Psi})-1), & \text{if } i^1 < i < i^2 \\ \phi m = \bar{\Psi}, & \text{if } i^2 \leq i \leq i^3 \\ i = \sigma(1-\eta\theta)(f'(\phi m)-1) + \sigma\eta(f'(\phi m + \bar{\Psi})-1), & \text{if } i^2 < i < i^3 \end{cases}$$

where i^1, i^2, and i^3 are respectively given by:

$$i^1 = \sigma(1-\eta\theta)(f'(k^* - \bar{\Psi})-1)$$
$$i^2 = \sigma(1-\eta\theta)(f'(\bar{\Psi})-1) + \sigma\eta\theta(f'(2\bar{\Psi})-1)$$
$$i^3 = \sigma(1-\eta\theta)(f'(\bar{\Psi})-1) + \sigma\eta(f'(2\bar{\Psi})-1)$$

□

Proof of Proposition 9. If $\bar{i}^1 < i \leq \bar{i}^2$, the allocation is given by Equations 31 and 32. Using the implicit function theorem on Eq. 32 to find:

$$\frac{\partial \hat{k}}{\partial i} = \frac{1}{\sigma(1-\eta\theta)f''(\hat{k}) + \sigma\eta\theta f''(k)} < 0 \tag{A.14}$$

$$\frac{\partial \hat{k}}{\partial \theta} = \frac{\eta(f'(\hat{k}) - f'(k))}{(1-\eta\theta)f''(\hat{k}) + \eta\theta f''(k)} \leq 0 \tag{A.15}$$

$$\frac{\partial \hat{k}}{\partial \eta} = \frac{\theta(f'(\hat{k}) - f'(k))}{(1-\eta\theta)f''(\hat{k}) + \eta\theta f''(k)} \leq 0 \tag{A.16}$$

$$\frac{\partial \hat{k}}{\partial \bar{\Psi}} = -\frac{\sigma\eta\theta f'(k)}{(1-\eta\theta)f''(\hat{k}) + \eta\theta f''(k)} \leq 0 \tag{A.17}$$

as $f'(\hat{k}) - f'(k) > 0$ and $f''(k) < 0$. Next, differentiate aggregate capital w.r.t. to i, θ, η, and $\bar{\Psi}$:

$$\frac{\partial K}{\partial i} = \sigma \frac{\partial \hat{k}}{\partial i}$$
$$\frac{\partial K}{\partial \theta} = \sigma \frac{\partial \hat{k}}{\partial \theta}$$
$$\frac{\partial K}{\partial \eta} = \sigma \bar{\Psi} + \sigma \frac{\partial \hat{k}}{\partial \eta}$$
$$\frac{\partial K}{\partial \bar{\Psi}} = \sigma \eta + \sigma \frac{\partial \hat{k}}{\partial \Psi}$$

If $\bar{i}^2 < i < \bar{i}^3$, the allocation is given by Eq. 33 and Eq. 34. The allocation is independent of i, θ, or η. Only η affects aggregate capital investment. Then the descriptive statistics are given by:

$$\frac{\partial K}{\partial \eta} = \sigma \bar{\Psi}$$
$$\frac{\partial \hat{k}}{\partial \bar{\Psi}} = 1$$
$$\frac{\partial k}{\partial \bar{\Psi}} = 2$$
$$\frac{\partial K}{\partial \bar{\Psi}} = \sigma(1 + \eta)$$

□

Proof of Proposition 10. From the proof of proposition 5, it only required that for any η the equilibrium allocation is given by the abundant liquidity case. As $\bar{i}^1$ is decreasing in η, this requires that $i \leq \sigma(1 - \bar{\eta}\theta)$. The rest of the proof follows from the proof of Proposition 5. □

Proof of Proposition 11. From the proof of Proposition 7, it is required that for any $\eta \in [0, \bar{\eta}]$ the equilibrium allocation is given by the scarce liquidity case. Therefore, $i > i^3$ for all $\eta \in [0, \bar{\eta}]$. Consequently, i should be sufficiently high as stated in

Proposition 7. The rest of the proof follows as before. □

B Nash Bargaining

In our paper, we used a Kalai proportional bargaining protocol. Now, we show that by using a Generalized Nash Bargaining protocol our results remain robust. In this case, the bargaining problem (Eq. 7-10) becomes:

$$(\psi_t, d_t, k_t) \in \arg\max_{\psi_t, d_t, k_t} \left\{ \left(f(k_t) - d_t + \phi_t \psi_t - k_t - f(\hat{k}_t) + \hat{k}_t \right)^{\theta} (d_t - \phi_t \psi_t)^{1-\theta} \right\}$$

subject to

$$\frac{k_t}{\phi_t} \leq m_t^P + \psi_t$$
$$-m_t^P \leq \psi_t \leq m_t^U$$

Recall that θ denotes the bargaining power of a productive bank. Solving gives the following F.O.C.s and complementary slackness conditions:

$$d_t = \phi_t \psi_t + (1-\theta)[f(k_t) - k_t - f(\hat{k}_t) + \hat{k}_t]$$
$$p_t \lambda^1 = \theta(f'(k_t) - 1) \left[\frac{d_t - \phi_t \psi_t}{f(k_t) - d_t + \phi_t \psi_t - k_t - f(\hat{k}_t) + \hat{k}_t} \right]^{1-\theta}$$
$$\lambda^1 = \lambda^2$$
$$\lambda^1 (m_t^P + \psi_t - \frac{k_t}{\phi_t}) = 0$$
$$\lambda_2 (m_t^U - \psi_t) = 0$$

where we used that $-m^P \leq \psi_t$ never binds due to the Inada conditions. If $\lambda_1 = \lambda_2 = 0$, the terms-of-trade are given by:

$$
\begin{aligned}
k_t &= k^* \\
\psi_t &\in \left(\frac{k^*}{\phi} - m_t^P, m_t^U\right) \\
d_t &= \phi_t \psi_t + (1-\theta)[f(k_t) - k_t - f(\hat{k}_t) + \hat{k}_t]
\end{aligned}
$$

whereas if both constraints bind, the terms-of-trade are given by:

$$
\begin{aligned}
k_t &= \phi_t(m_t^U + m_t^P) \\
\psi_t &= m_t^U \\
d_t &= \phi_t \psi_t + (1-\theta)[f(k_t) - k_t - f(\hat{k}_t) + \hat{k}_t]
\end{aligned}
$$

Therefore, the resulting term-of trade are equivalent to proportional Kalai bargaining.

C Interior Matching Efficiency: Debt Constrained Equilibrium

Proposition 12. *For $\sigma < \frac{1}{2}$ and θ above a threshold, there is a range of $\{i, \bar{\Psi}\}$ for which $\eta^* < \bar{\eta}$.*

Proof of Proposition 12. First, note that from Equations 37 and 38 both $\bar{i}^1$ and $\bar{i}^2$ are (weakly) decreasing in η . Therefore, to have a debt constrained equilibrium given by Equations 31 and 32 for all $\eta \in [0, \bar{\eta}]$, we require that:

$$\bar{i}^1(0) < i < \bar{i}^2(\bar{\eta}).$$

Using Equations 37 and 38 this can be re-written to:

$$\sigma f'(k^* - \bar{\Psi}) < \sigma(1 - \bar{\eta}\theta)(f'(\bar{\Psi}) - 1) + \sigma\bar{\eta}\theta(f'(2\bar{\Psi}) - 1)$$

where the LHS is increasing in $\bar{\Psi}$ and the RHS is decreasing. As $\bar{\Psi} \to 0$ the LHS is positive and bounded, while the RHS goes to ∞. Similarly, as $\bar{\Psi} \to k^*$ gives that LHS goes to ∞, while RHS is negative and bounded. Hence, there exists a $\bar{\Psi}$ for which the LHS and RHS are equal and for every $\bar{\Psi}$ below the inequality is satisfied.

The derivative of welfare (Eq. 23) w.r.t. to η is then given by:

$$\begin{aligned}\frac{(1-\beta)}{\sigma}\frac{\partial \mathcal{W}}{\partial \eta} &= f(k) - \bar{\Psi} - f(\hat{k}) \\ &+ \Big(\eta(f'(k) - 1) + (1-\eta)(f'(\hat{k}) - 1)\Big)\frac{\theta(f'(\hat{k}) - f'(k))}{(1-\theta\eta)f''(\hat{k}) + \theta\eta f''(k)}\end{aligned}$$

where we used Eq A.16. Define $\rho k^* = \bar{\Psi}$, where from Proposition 8 $\rho < \frac{1}{2}$, and ω as $\omega\hat{k} = k^*$, where ω depends on i, η, and θ. For simplicity, we suppress this dependence. In addition, note that the equilibrium allocation (Eq. 31 and Eq. 32) implies that $\omega \in (\rho, 1-\rho)$. Using that the properties of the homogeneous function imply that $\alpha f(k^*) = k^*$, the derivative becomes:

$$\begin{aligned}\frac{(1-\beta)}{\sigma}\frac{\partial \mathcal{W}}{\partial \eta} &= f(k^*)\Bigg((\omega+\rho)^\alpha - \omega^\alpha - \alpha\rho \\ &- \frac{\theta\alpha}{(1-\alpha)}\big(\eta(\rho+\omega)^{\alpha-1} + (1-\eta)\omega^{\alpha-1} - 1\big)\frac{(\omega^{\alpha-1} - (\rho+\omega)^{\alpha-1})}{(1-\eta\theta)\omega^{\alpha-2} + \eta(\rho+\omega^{\alpha-2})}\Bigg)\end{aligned}$$

Evaluate at $\eta = \bar{\eta} = 1$:

$$\begin{aligned}\frac{(1-\beta)}{\sigma}\frac{\partial \mathcal{W}}{\partial \eta}\bigg|_{\eta=1} &= f(k^*)\Bigg((\omega+\rho)^\alpha - \omega^\alpha - \alpha\rho \\ &- \frac{\theta\alpha}{(1-\alpha)}\big((\rho+\omega)^{\alpha-1} - 1\big)\frac{(\omega^{\alpha-1} - (\rho+\omega)^{\alpha-1})}{(1-\theta)\omega^{\alpha-2} + (\rho+\omega)^{\alpha-2}}\Bigg)\end{aligned}$$

If $\omega = 1-\rho$, this is equal to $f(k^*)(1 - (1-\rho)^\alpha - \alpha\rho)$, which is strictly positive as it is increasing in ρ and at $\rho = 0$ is positive. Similarly, if $\omega = \rho$ then the equation becomes

$$\rho^{\alpha}(2^{\alpha}-1)-\alpha\rho-\frac{\theta\alpha}{(1-\alpha)}\frac{(1-2^{\alpha-1})}{(1-\theta+2^{\alpha-2})}\left(2^{\alpha-1}\rho^{\alpha}-\rho\right)$$

The derivative w.r.t. ρ is:

$$\alpha[(2^{\alpha}-1)\rho^{\alpha-1}-1]-\frac{\theta\alpha}{(1-\alpha)}\frac{(1-2^{\alpha-1})}{(1-\theta+2^{\alpha-2})}(\alpha 2^{\alpha-1}\rho^{\alpha-1}-1)$$

Note that this is decreasing in ρ if

$$\theta>\frac{(2^{\alpha}-1)(1-\alpha)(1+2^{\alpha-2})}{(2^{\alpha}-1)(1-\alpha)+\alpha(1-2^{\alpha-1})2^{\alpha-1}}=\bar{\theta}$$

which is strictly below one for $\alpha\in(0,1)$. Therefore, if θ is above a threshold there exists a ρ that minimizes the original equation. If $\rho=0$, then the equation is zero and the first derivative goes to minus ∞. Consequently, for a range of ρ or equivalently $\bar{\Psi}$, the derivative is negative at $\omega=\rho$. As the derivative is continuous in ω, positive at $\omega=1-\rho$, and negative at $\omega=\rho$, there is range of ω for which the derivative is negative if $\theta>\bar{\theta}$. □